Chapter 9 Polar Coordinates and Complex Numbers

9-1 Polar Coordinates

Pages 557–558 Check for Understanding

1. There are infinitely many ways to represent the angle θ. Also, r can be positive or negative.

2. Draw the angle θ in standard position. Extend the terminal side of the angle in the opposite direction. Locate the point that is $|r|$ units from the pole along this extension.

3. Sample answer: $-60°$ and $300°$
Plot (4, 120) such that θ is in standard position and $|r|$ is 4 units from the pole. Extend the terminal side of the angle in the opposite direction. Locate the point that is 4 units from the pole along this extension.
$r = -4$
$\theta = 120 - 180$ or $\theta = 120 + 180$
$= -60 \qquad = 300$

4. The points 3 units from the origin in the opposite direction are on the circle where $r = 3$.

5. All ordered pairs of the form (r, θ) where $r = 0$.

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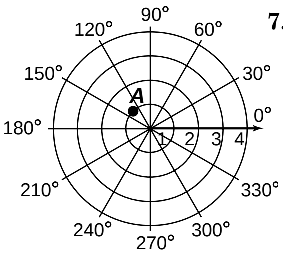

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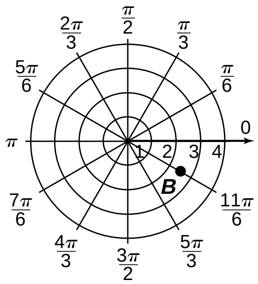

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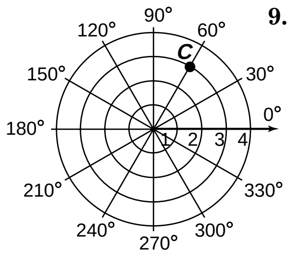

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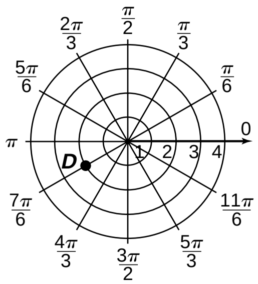

10. Sample answer: $\left(-2, \frac{13\pi}{7}\right), \left(-2, \frac{25\pi}{6}\right), \left(2, \frac{7\pi}{6}\right), \left(2, \frac{19\pi}{6}\right)$

$(r, \theta + 2k\pi)$

$\to \left(-2, \frac{\pi}{6} + 2(1)\pi\right) \to \left(-2, \frac{13\pi}{6}\right)$

$\to \left(-2, \frac{\pi}{6} + 2(2)\pi\right) \to \left(-2, \frac{25\pi}{6}\right)$

$(-r, \theta + (2k + 1)\pi)$

$\to \left(2, \frac{\pi}{6} + (1)\pi\right) \to \left(2, \frac{7\pi}{6}\right)$

$\to \left(2, \frac{\pi}{6} + (3)\pi\right) \to \left(2, \frac{19\pi}{6}\right)$

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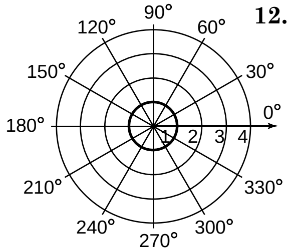

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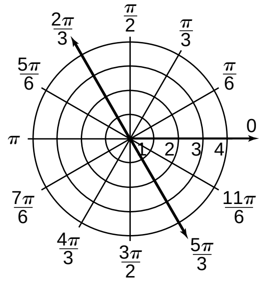

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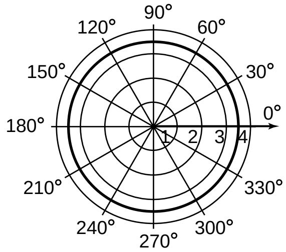
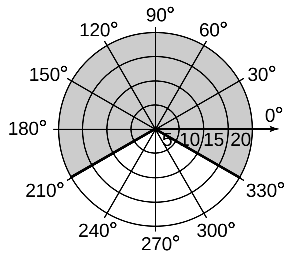


14. $P_1P_2 = \sqrt{2.5^2 + (-3)^2 - 2(2.5)(-3)\cos\left(-\frac{\pi}{4} - \frac{\pi}{6}\right)}$
$= \sqrt{6.25 + 9 + 15\cos\left(-\frac{5\pi}{12}\right)}$
$= \sqrt{5.25 + 15\cos\left(-\frac{5\pi}{12}\right)}$
≈ 4.37

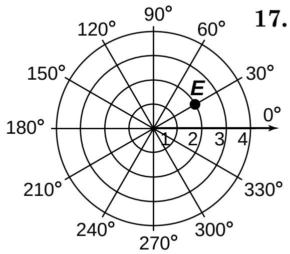
15a.



15b. $210 - (-30) = 240$
$A = \frac{N}{360}(\pi r^2)$
$= \frac{240}{360}(\pi 20^2)$
$\approx 838 \text{ ft}^2$

Pages 558–560 Exercises

16.



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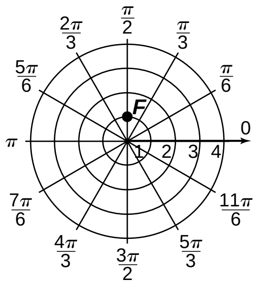

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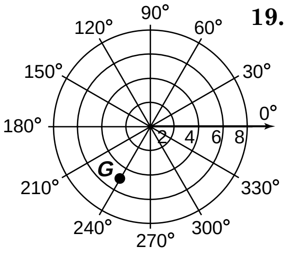

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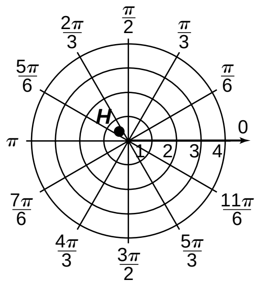

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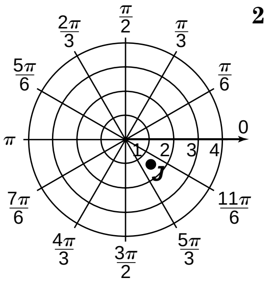

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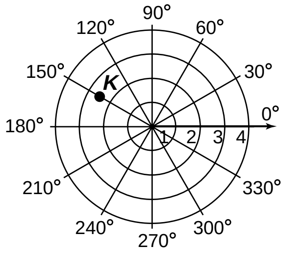

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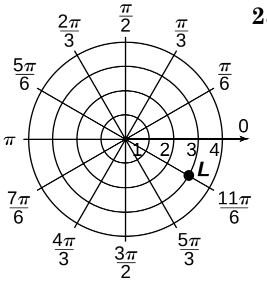

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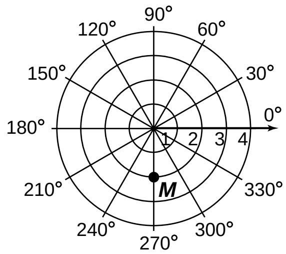

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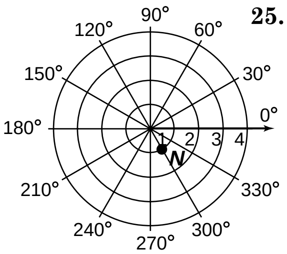

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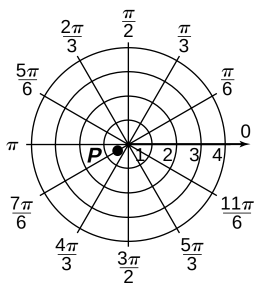

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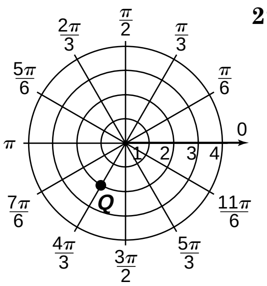

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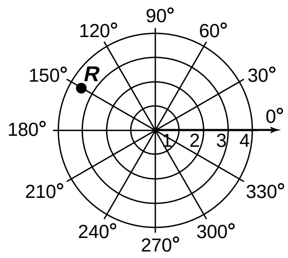


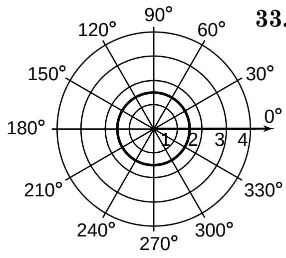
28. Sample answer: $\left(2, \frac{\pi}{3}\right), \left(2, \frac{7\pi}{3}\right), (-2, 240°),$
$(-2, 600°)$
$(r, \theta + 2k\pi)$
$\rightarrow \left(2, \frac{\pi}{3} + 2(0)\pi\right) \rightarrow \left(2, \frac{\pi}{3}\right)$
$\rightarrow \left(2, \frac{\pi}{3} + 2(1)\pi\right) \rightarrow \left(2, \frac{7\pi}{3}\right)$
$(-r, \theta + (2k + 1)180°)$
$\rightarrow (-2, 60° + (1)180°) \rightarrow (-2, 240°)$
$\rightarrow (-2, 60° + (3)180°) \rightarrow (-2, 600°)$

29. Sample answer: $(1.5, 540°), (1.5, 900°), (-1.5, 0°),$
$(-1.5, 360°)$
$(r, \theta + 360k°)$
$\rightarrow (1.5, 180° + 360(1)°) \rightarrow (1.5, 540°)$
$\rightarrow (1.5, 180° + 360(2)°) \rightarrow (1.5, 900°)$
$(-r, \theta + (2k + 1)180°)$
$\rightarrow (-1.5, 180° + (-1)180°) \rightarrow (-1.5, 0°)$
$\rightarrow (-1.5, 180° + (1)180°) \rightarrow (-1.5, 360°)$

30. Sample answer: $\left(-1, \frac{7\pi}{3}\right), \left(-1, \frac{13\pi}{3}\right), \left(1, \frac{4\pi}{3}\right),$
$\left(1, \frac{10\pi}{3}\right)$
$(r, \theta + 2k\pi)$
$\rightarrow \left(-1, \frac{\pi}{3} + 2(1)\pi\right) \rightarrow \left(-1, \frac{7\pi}{3}\right)$
$\rightarrow \left(-1, \frac{\pi}{3} + 2(2)\pi\right) \rightarrow \left(-1, \frac{13\pi}{3}\right)$
$(-r, \theta + (2k + 1)\pi)$
$\rightarrow \left(1, \frac{\pi}{3} + (1)\pi\right) \rightarrow \left(1, \frac{4\pi}{3}\right)$
$\rightarrow \left(1, \frac{\pi}{3} + (3)\pi\right) \rightarrow \left(1, \frac{10\pi}{3}\right)$

31. Sample answer: $(4, 675°), (4, 1035°), (-4, 135°),$
$(-4, 495°)$
$(r, \theta + 360k°)$
$\rightarrow (4, 315 + 360(1)°) \rightarrow (4, 675°)$
$\rightarrow (4, 315 + 360(2)°) \rightarrow (4, 1035°)$
$(-r, \theta + (2k + 1)180°)$
$\rightarrow (-4, 315 + (-1)180°) \rightarrow (-4, 135°)$
$\rightarrow (-4, 315 + (1)180°) \rightarrow (-4, 495°)$

32.



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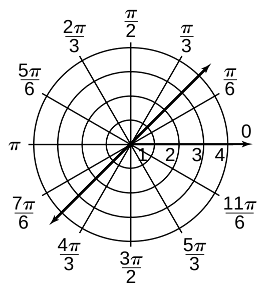

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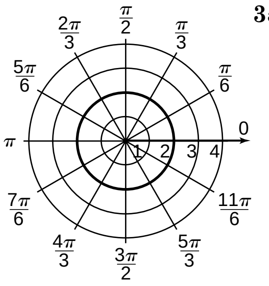

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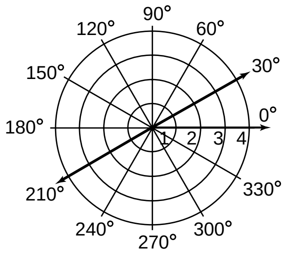

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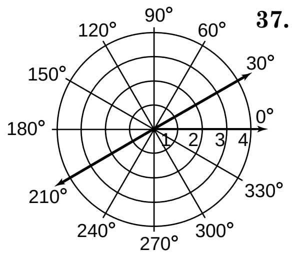

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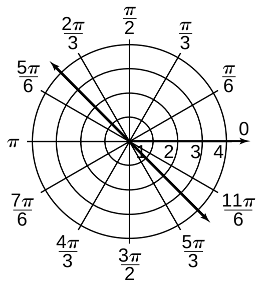

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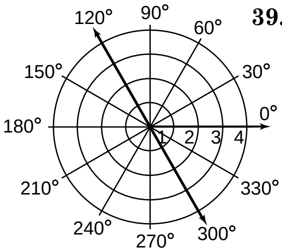

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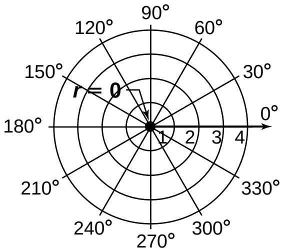

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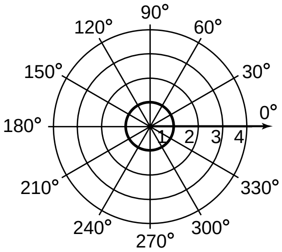


41. $r = \sqrt{2}$ or $r = -\sqrt{2}$ for any θ.

42. $P_1P_2 = \sqrt{4^2 + 6^2 - 2(4)(6)\cos(105° - 170°)}$
$= \sqrt{16 + 36 - 48\cos(-65°)}$
$= \sqrt{52 - 48\cos(-65)}$
≈ 5.63

43. $P_1P_2 = \sqrt{1^2 + 5^2 - 2(1)(5)\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)}$
$= \sqrt{1 + 25 - 10\cos\left(\frac{7\pi}{12}\right)}$
$= \sqrt{26 - 10\cos\left(\frac{7\pi}{12}\right)}$
≈ 5.35

44. $P_1P_2 =$
$\sqrt{(-2.5)^2 + (-1.75)^2 - 2(-2.5)(-1.75)\cos\left(-\frac{2\pi}{5} - \frac{\pi}{8}\right)}$
$= \sqrt{6.25 + 30.0625 - 8.75\cos\left(-\frac{21\pi}{40}\right)}$
$= \sqrt{9.3125 - 8.75\cos\left(-\frac{21\pi}{40}\right)}$
≈ 3.16

45. $P_1P_2 = \sqrt{1.3^2 + (-3.6)^2 - 2(1.3)(-3.6)\cos(-62° - (-47°))}$
$= \sqrt{1.69 + 12.96 + 9.36\cos(-62° + 47°)}$
$= \sqrt{14.65 + 9.36\cos(-15°)}$
≈ 4.87

46. $r = \sqrt{(-3)^2 + 4^2} = 5$
$\sin\theta = \frac{4}{5}, \theta \approx 53°$
$180° - 53° = 127°$
Sample answer: (5, 127°)

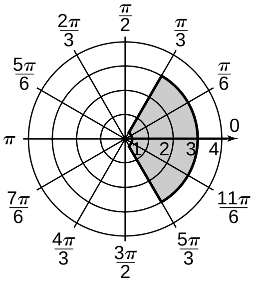
47. There are 360° in a circle. If the circle is cut into 6 equal pieces, each slice measures $\frac{360}{6}$ or 60°. Beginning at the origin, the equation of the first line is $\theta = 0°$. The equation of the next line, rotating counterclockwise, is $\theta = 0 + 60$ or 60°. The equation of the last line is $\theta = 60 + 60$ or 120°. Note that lines extend through the origin, so 3 lines create 6 pieces.

48. $P_1P_2 = \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta - \theta)}$
$= \sqrt{r_1{}^2 - r_2{}^2 - 2r_1r_2\cos 0}$
$= \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2}$
$= \sqrt{(r_1 - r_2)^2}$
$= |r_1 - r_2|$

49a. When $\theta = 120°$, $r = 17$. The maximum speed at 120° is 17 knots.

49b. When $\theta = 150°$, $r = 13$. The maximum speed at 150° is 13 knots.

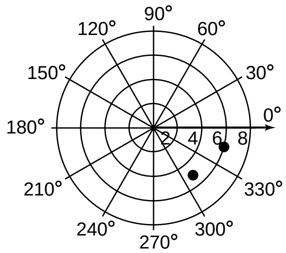
50a.



50b. $\frac{\pi}{3} - \left(-\frac{\pi}{3}\right) = \frac{2\pi}{3}$ or 120°
Let $R = 3 \cdot 100$ or 300 and let $r = 0.25 \cdot 100$ or 25.
$A = \frac{N}{360}(\pi R^2) - \frac{N}{360}(\pi r^2)$
$= \frac{120}{360}(\pi(300^2) - \frac{120}{360}(\pi(25)^2)$
$= \frac{120}{360}\pi(90{,}000 - 625)$
$\approx 93{,}593 \text{ ft}^2$
If each person's seat requires 6 ft^2 of space, there are $\frac{93{,}593}{6}$ or 15,599 seats.

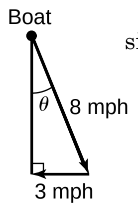
51. The distance formula is symmetric with respect to (r_1, θ_1) and (r_2, θ_2). That is,
$\sqrt{r_2{}^2 + r_1{}^2 - 2r_2r_1\cos(\theta_1 - \theta_2)}$
$= \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2\cos[-(\theta_2 - \theta_1)]}$
$= \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

52a.



52b. $P_1P_2 = \sqrt{5^2 + 6^2 - 2(5)(6)\cos(345° - 310°)}$
$= \sqrt{25 + 36 - 60\cos(35°)}$
$= \sqrt{61 - 60\cos(35°)}$
≈ 3.44
No; the planes are 3.44 miles apart.

53. Draw a picture.



$\sin\theta = \frac{3}{8}$
$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{3}{8}\right)$
$\theta \approx 22.0°$

54. $\langle 3, -2, 4\rangle \cdot \langle 1, -4, 0\rangle = (3)(1) + (-2)(-4) + (4)(0)$
$= 3 + 8 + 0$
$= 11$
No, the vectors are not perpendicular because their inner product is not 0.

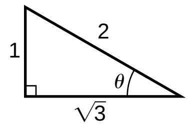
55. Rewrite $y = 9x - 3$ as $9x - y - 3 = 0$.

$$\begin{aligned} d &= \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}} \\ &= \frac{9(-3) + (-1)(2) + (-3)}{\pm\sqrt{9^2 + (-1)^2}} \\ &= \frac{-32}{\pm\sqrt{82}} \\ &= \frac{-32}{\pm\sqrt{82}} \cdot \frac{\sqrt{82}}{\sqrt{82}} \\ &= \pm\frac{-32\sqrt{82}}{82} \\ &= \frac{16\sqrt{82}}{41} \qquad \text{Distance is always positive.} \end{aligned}$$

56. $$\begin{aligned} \frac{1 - \sin^2\alpha}{\sin^2\alpha} &= \frac{1}{\sin^2\alpha} - 1 \\ &= \csc^2\alpha - 1 \\ &= \cot^2\alpha \end{aligned}$$

57. Arc cos $\frac{\sqrt{3}}{2} = 30°$

In a 30°-60°-90° right triangle, the angle opposite the smallest leg is 30°.



58. $y = 5 \cos 4\theta$

Amplitude = 5; Period = $\frac{2\pi}{4}$ or $\frac{\pi}{2}$

59. $b \sin A = 18.6 \sin 30°$
$= 9.3$

Since $a = b \sin A$, there is one solution.

Find B.

$$\frac{18.6}{\sin B} = \frac{9.3}{\sin 30°}$$
$$18.6 \sin 30° = 9.3 \sin B$$
$$\frac{18.6 \sin 30°}{9.3} = \sin B$$
$$90° \approx B$$

Find C.

$$C = 180° - 90° - 30°$$
$$= 60°$$

Find c.

$$\frac{c}{\sin 60°} = \frac{9.3}{\sin 30°}$$
$$c \sin 30° = 9.3 \sin 60°$$
$$c = \frac{9.3 \sin 60°}{\sin 30°}$$
$$c = 16.1$$

60. 3 or 1 positive

$f(-x) = -x^3 - 4x^2 - 4x - 1$

0 negative

$\frac{P}{Q}: \pm 1$

Since there are only positive real zeros, the only rational real zero is 1.

61.
$$\begin{array}{r} x - 3 \\ x + 5\overline{)x^2 + 2x - 3} \\ \underline{x^2 + 5x} \\ -3x - 3 \\ \underline{-3x - 15} \\ 10 \end{array}$$

As $x \to \pm\infty$, $\frac{10}{x + 5} \to 0$. Therefore, the slant asymptote is $y = x - 3$.

62. y-axis:

For x: $f(x) = x^4 - 3x^2 + 2$

For $-x$: $f(-x) = (-x)^4 - 3(-x)^2 + 2$
$= x^4 - 3x^2 + 2$

So, in general, point $(-x, y)$ is on the graph if and only if (x, y) is on the graph.

63. $$\begin{aligned} \begin{vmatrix} -2 & 4 & -1 \\ 1 & -1 & 0 \\ -3 & 4 & 5 \end{vmatrix} &= -2\begin{vmatrix} -1 & 0 \\ 4 & 5 \end{vmatrix} - 4\begin{vmatrix} 4 & 0 \\ -3 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -1 \\ -3 & 4 \end{vmatrix} \\ &= -2(-5) - 4(5) - 1(1) \\ &= -11 \end{aligned}$$

64. $11 - (-3) = 14$
$11 - (-2) = 13$
$11 - (-1) = 12$
$11 - 0 = 11$
$\{(-3, 14), (-2, 13), (-1, 12), (0, 11)\}$
For each x-value, there ia a unique v-value.
Yes, the relation is a function.

65. Since the two triangles formed are right triangles, the side opposite the right angles, $\overline{AB}$, intercept an arc measuring 180°, or half the circle. $\overline{AB}$ is a diameter.

$C = \pi d$
$50\pi = \pi d$
$50 = d$

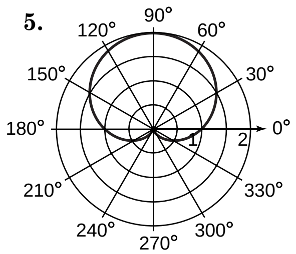
The correct choice is E.

9-2 Graphs of Polar Equations

Page 565 Check for Understanding

1. Sample answer: $r = \sin 2\theta$
   The graph of a polar equation whose form is $r = a \cos n\theta$ or $a \sin n\theta$, where n is a positive integer, is a rose.
2. $-1 \leq \sin \theta \leq 1$ for any value of θ. Therefore, the maximum value of $r = 3 + 5 \sin \theta$ is $r = 3 + 5(1)$ or 8. Likewise, the minimum value of $r = 3 + 5 \sin \theta$ is $r = 3 + 5(-1)$ or -2.
3. The polar coordinates of a point are not unique. A point of intersection may have one representation that satisfies one equation in a system, another representation that satisfies the other equation, but no representation that satisfies both simultaneously.
4. Barbara is correct. The interval $0 \leq \theta \leq \pi$ is not always sufficient. For example, the interval $0 \leq \theta \leq \pi$ only generates two of the four petals for the rose $r = \sin 2\theta$. $r = \sin \frac{\theta}{2}$ is an example where values of θ from 0 to 4π would have to be considered.

5.



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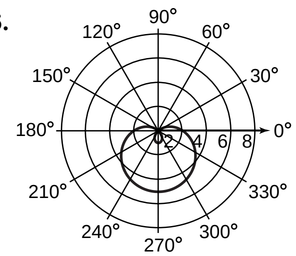
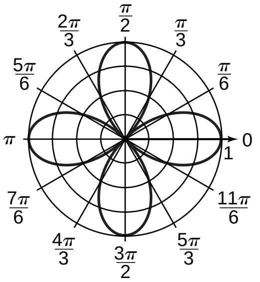
6.



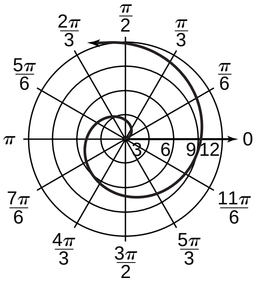
limaçon

7.



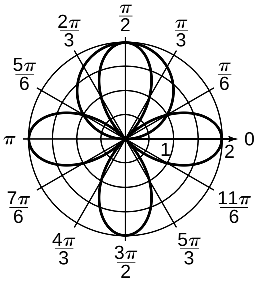
rose

8.



spiral of Archimedes

9.



$$
\begin{aligned}
2\sin\theta &= 2\cos 2\theta\\
\sin\theta &= \cos 2\theta\\
\sin\theta &= 1 - 2\sin^2\theta\\
2\sin^2\theta + \sin\theta - 1 &= 0\\
(2\sin\theta - 1)(\sin\theta + 1) &= 0
\end{aligned}
$$

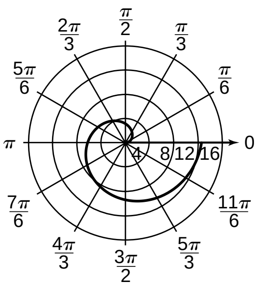
$2\sin\theta - 1 = 0$ or $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ $\qquad\sin\theta = -1$

$\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$ or $\theta = \frac{3\pi}{2}$

If $\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$ is substituted in either original equation, $r = 1$. If $\theta = \frac{3\pi}{2}$ is substituted in either original equation, $r = -2$. The solutions are $\left(1, \frac{\pi}{6}\right)$, $\left(1, \frac{5\pi}{6}\right)$, and $\left(-2, \frac{3\pi}{2}\right)$.

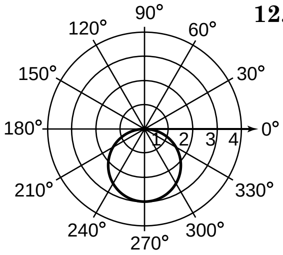
10a.



10b. Sample answer: $0 \le \theta \le \frac{14\pi}{3}$
Begin at the origin and "spiral" twice around it, or through 4π radians. Move straight up through $4\pi + \frac{\pi}{2}$ or $\frac{9\pi}{2}$ radians. Now move to the left slightly, through approximately $\frac{9\pi}{2} + \frac{\pi}{6}$ or $\frac{14\pi}{3}$ radians.

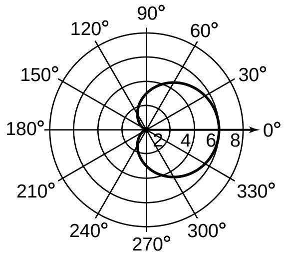
Pages 565–567 Exercises

11.



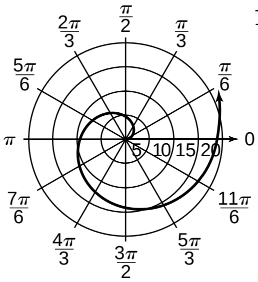
circle

12.



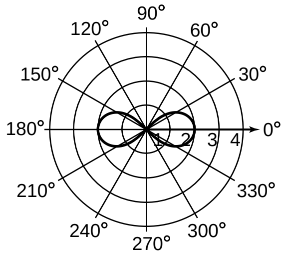
cardioid

13.



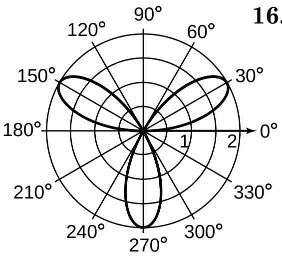
spiral of Archimedes

14.



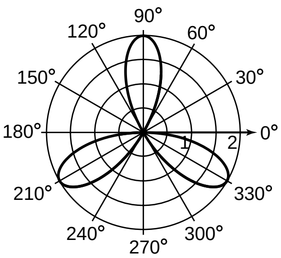
lemniscate

15.



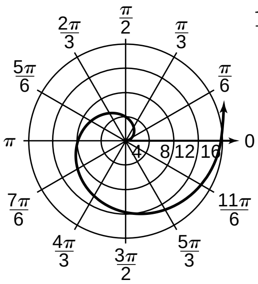
rose

16.



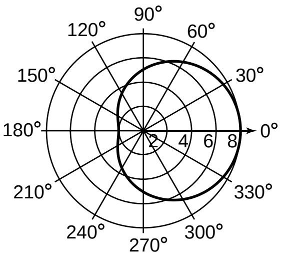
rose

17.



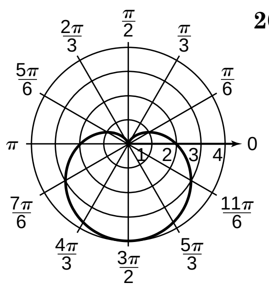
Spiral of Archimedes

18.



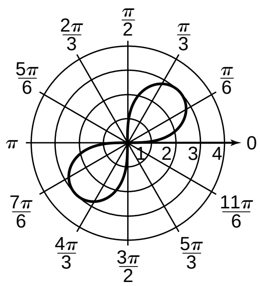
limaçon

19.



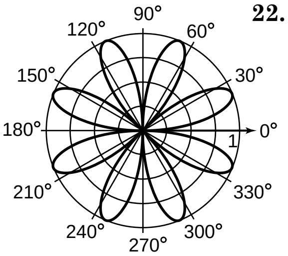
cardioid

20.



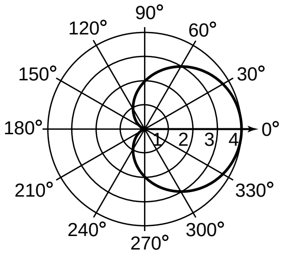
lemniscate

21.



rose

22.



cardioid

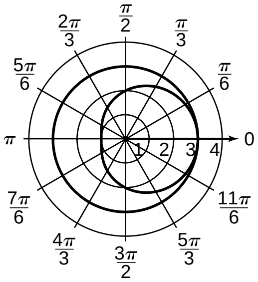
23. Sample answer: $r = \sin 3\theta$
The graph of a polar equation of the form $r = a \cos 3\theta$ or $r = a \sin 3\theta$ is a rose with 3 petals.

24. Sample answer: $r = \frac{\theta}{2}$

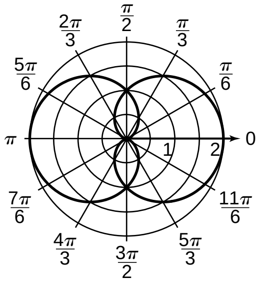
$\frac{\pi}{4} = a\left(\frac{\pi}{2}\right)$ $\qquad r = \frac{1}{2}\theta$

$\frac{1}{2} = a$ $\qquad r = \frac{\theta}{2}$

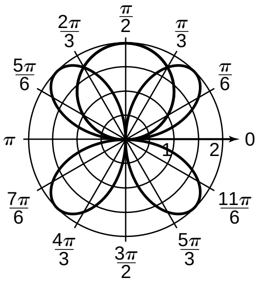
25. $3 = 2 + \cos \theta$
$1 = \cos \theta$
$\theta = 0$
The solution is (3, 0)



26. $1 + \cos \theta = 1 - \cos \theta$
$2 \cos \theta = 0$
$\cos \theta = 0$
$\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$
Substituting each angle into either of the original equations gives $r = 1$, so the solutions of the system are $\left(1, \frac{\pi}{2}\right)$ and $\left(1, \frac{3\pi}{2}\right)$.



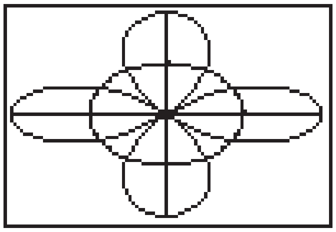
27.



$2 \sin \theta = 2 \sin 2\theta$
$\sin \theta = \sin 2\theta$
$\sin \theta = 2 \cos \theta \sin \theta$
$0 = 2 \cos \theta \sin \theta - \sin \theta$
$0 = \sin \theta\,(2 \cos \theta - 1)$
$\sin \theta = 0$ or $2 \cos \theta - 1 = 0$
$\cos \theta = \frac{1}{2}$
$\theta = 0$ or π or $\theta = \frac{\pi}{3}$ or $\theta = \frac{5\pi}{3}$

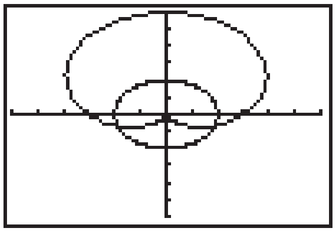
If $\theta = 0$ or $\theta = \pi$ is substituted in either original equation, $r = 0$. If $\theta = \frac{\pi}{3}$ or $\theta = \frac{5\pi}{3}$ is substituted in either original equation, $r = \sqrt{3}$ or $r = -\sqrt{3}$, respectively. The solutions are (0, 0), (0, π), $\left(\sqrt{3}, \frac{\pi}{3}\right)$, and $\left(-\sqrt{3}, \frac{5\pi}{3}\right)$.

28. (1, 0.5), (1, 1.0), (1, 2.1), (1, 2.6), (1, 3.7), (1, 4.2), (1, 5.2), (1, 5.8)



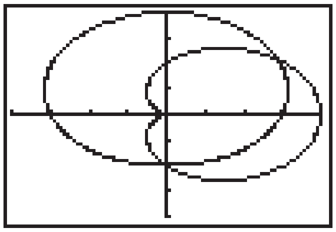
[−2, 2] scl: 1 by [−2, 2] scl: 1

29. (2, 3.5), (2, 5.9)



[−6, 6] scl:1 by [−6, 6] scl: 1

30. (3.6, 0.6), (2.0, 4.7)

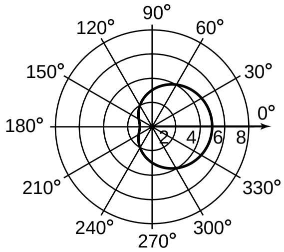


[−4, 4] scl: 1 by [−4, 4] scl: 1

31a. If the lemniscate is 6 units from end to end, then $a = \frac{1}{2}(6)$ or 3.
$r^2 = 9 \cos 2\theta$ or $r^2 = 9 \sin 2\theta$

31b. If the lemniscate is 8 units from end to end, then $a = \frac{1}{2}(8)$ or 4.
$r^2 = 16 \cos 2\theta$ or $r^2 = 16 \sin 2\theta$

32.

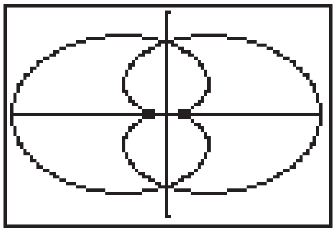


This microphone will pick up more sounds from behind than the cardioid microphone.

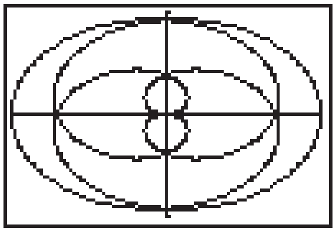
33. $0 \le \theta \le 4\pi$: Begin at the origin and curl around once, or through 2π radians. Curl around a second time and go through $2\pi + 2\pi$ or 4π radians.

34. All screens are [−1, 1] scl: 1 by [−1, 1] scl: 1

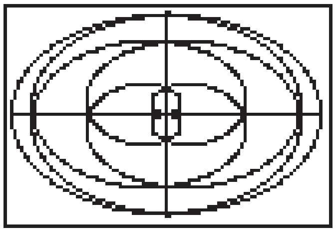
34a. $r = \cos \frac{\theta}{2}$



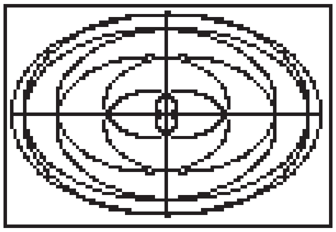
$r = \cos \frac{\theta}{4}$



$r = \cos \frac{\theta}{6}$

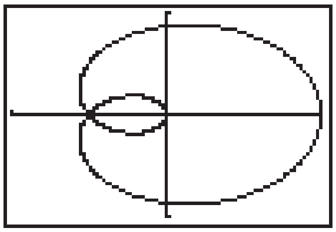


$r = \cos \frac{\theta}{8}$

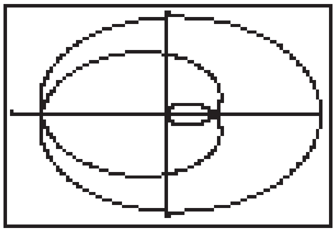


When $n = 10$, two more outer rings will appear.

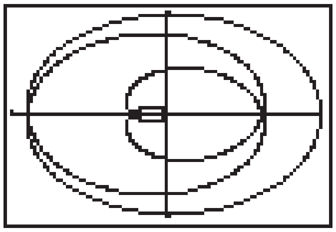
34b. $r = \cos \frac{\theta}{3}$



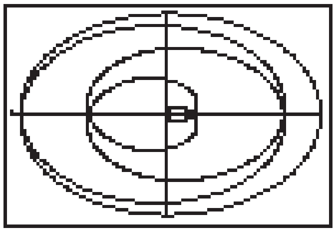
$r = \cos \frac{\theta}{5}$



$r = \cos \frac{\theta}{7}$



$r = \cos \frac{\theta}{9}$



When $n = 11$, the innermost loop will be on the left and there will be an additional outer ring.

35. Sample answer: $r = -1 - \sin \theta$
A heart resembles the shape of a cardioid. The sine function orients the heart so that the axis of symmetry is along the y-axis. If $a = -1$, the heart points in the right direction.

36a. For a limaçon to go back on itself and have an inner loop, r must change sign. This will happen if $|b| > |a|$.

36b. For the other two cases, $|a| \geq |b|$. Experimentation shows that the dimple disappears when $|a| = |2b|$, so there is a dimple if $|b| \leq |a| < |2b|$.

36c. For this remaining case, there is neither an inner loop nor a dimple if $|a| \geq |2b|$.

37a. Subtracting α from θ rotates the graph counterclockwise by an angle of α.

37b. Multiplying θ by -1 reflects the graph about the polar axis or x-axis.

37c. Multiplying the function by -1 changes r to its opposite, so the graph is reflected about the origin.

37d. Multiplying the function by c results in a dilation by a factor of c. (Points on the graph move closer to the origin if $c < 1$, or farther away from the origin if $c > 1$.)

38. Sample answer: (4, 405°), (4, 765°), (−4, −135°), (−4, 225°)
$(r, \theta + 360k°)$
$\rightarrow (4, 45° + 360(1)°) \rightarrow (4, 405°)$
$\rightarrow (4, 45° + 360(2)°) \rightarrow (4, 765°)$
$(r, \theta + (2k + 1)180°)$
$\rightarrow (-4, 45° + (-1)180°) \rightarrow (-4, -135°)$
$\rightarrow (-4, 45° + (1)180°) \rightarrow (-4, 225°)$

39. $\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & 2 & 4 \end{vmatrix}$

$= \begin{vmatrix} 3 & 0 \\ 2 & 4 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} \vec{k}$

$= 12\vec{i} - 8\vec{j} + 7\vec{k}$

$= \langle 12, -8, 7 \rangle$

$\langle 2, 3, 0 \rangle \cdot \langle 12, -8, 7 \rangle = 24 + (-24) + 0$ or 0

$\langle -1, 2, 4 \rangle \cdot \langle 12, -8, 7 \rangle = -12 + (-16) + 28$ or 0

40. 3.5 cm, 87°

41. $\frac{\sin^2 x}{\cos^4 x + \cos^2 x \sin^2 x} \stackrel{?}{=} \tan^2 x$

$\frac{\sin^2 x}{\cos^2 x (\cos^2 x + \sin^2 x)} \stackrel{?}{=} \tan^2 x$

$\frac{\sin^2 x}{(\cos^2 x)(1)} \stackrel{?}{=} \tan^2 x$

$\frac{\sin^2 x}{\cos^2 x} \stackrel{?}{=} \tan^2 x$

$\tan^2 x = \tan^2 x$

42. Find C.

$C = 180° - 21°15' - 49°40'$
$= 109°5'$

Find b.

$\frac{b}{\sin 49°40'} = \frac{28.9}{\sin 109°5'}$

$b \sin 109°5' = 28.9 \sin 49°40'$

$b = \frac{28.9 \sin 49°40'}{\sin 109°5'}$

$b \approx 23.3$

Find a.

$\frac{a}{\sin 21°15'} = \frac{28.9}{\sin 109°5'}$

$a \sin 109°5' = 28.9 \sin 21°15'$

$a = \frac{28.9 \sin 21°15'}{\sin 109°5'}$

$a \approx 11.1$

43.
$$\begin{array}{l} \\ \text{Bus} \\ \text{Train} \end{array} \begin{array}{c} \begin{array}{ccc} \text{NY} & \text{LA} & \text{Miami} \end{array} \\ \begin{bmatrix} \$240 & \$199 & \$260 \\ \$254 & \$322 & \$426 \end{bmatrix} \end{array}$$

44. $\frac{1}{8} + \frac{6}{4} = \frac{1}{8} + \frac{12}{8} = \frac{13}{8}$

So $\frac{\frac{1}{8} + \frac{6}{4}}{\frac{3}{16}} = \frac{\frac{13}{8}}{\frac{3}{16}} = \frac{13}{8} \cdot \frac{16}{3}$

$= \frac{26}{3}$

The correct choice is A.

9-3 Polar and Rectangular Coordinates

Page 571 Check for Understanding

1. Sample answer: $(2\sqrt{2}, 45°)$

$r = \sqrt{2^2 + 2^2}$ $\quad\theta = \text{Arctan}\left(\frac{2}{2}\right)$

$= \sqrt{8}$ $\quad = 45°$

$= 2\sqrt{2}$

2. The quadrant that the point lies in determines whether θ is given by Arctan $\frac{y}{x}$ or Arctan $\frac{y}{x} + \pi$.

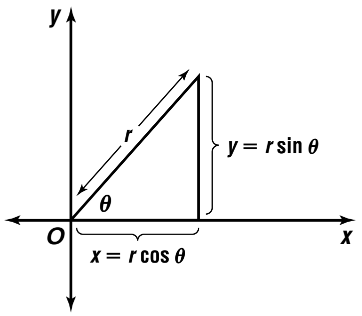
3. $x = 2$

$r \cos \theta = 2$

$r = \frac{2}{\cos \theta}$

$r = 2 \sec \theta$

4. To convert from polar coordinates to rectangular coordinates, substitute r and θ into the equations $x = r \cos \theta$ and $y = r \sin \theta$. To convert from rectangular coordinates to polar coordinates, use the equation $r = \sqrt{x^2 + y^2}$ to find r. If $x > 0$, $\theta =$ Arctan $\frac{y}{x}$. If $x < 0$, $\theta =$ Arctan $\frac{y}{x} + \pi$. If $x = 0$, you can use $\frac{\pi}{2}$ or any coterminal angle for θ.



5. $r = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2}$ $\quad\theta = \text{Arctan}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right)$

$= \sqrt{4}$ or 2 $\quad = \frac{3\pi}{4}$

$\left(2, \frac{3\pi}{4}\right)$

6. $r = \sqrt{(-2)^2 + (-5)^2}$ $\quad\theta = \text{Arctan}\left(\frac{-5}{-2}\right) + \pi$

$= \sqrt{29} \approx 5.39$ $\quad \approx 4.33$

$(5.39, 4.33)$

7. $x = -2 \cos\left(\frac{4\pi}{3}\right)$ $\quad y = -2 \sin\left(\frac{4\pi}{3}\right)$

$= 1$ $\quad = \sqrt{3}$

$(1, \sqrt{3})$

8. $x = 2.5 \cos 250°$ $\quad y = 2.5 \sin 250°$

≈ -0.86 $\quad = -2.35$

$(-0.86, -2.35)$

9. $y = 2$

$r \sin \theta = 2$

$r = \frac{2}{\sin \theta}$

$r = 2 \csc \theta$

10. $x^2 + y^2 = 16$

$(r \cos \theta)^2 + (r \sin \theta)^2 = 16$

$r^2(\cos^2 \theta + \sin^2 \theta) = 16$

$r^2 = 16$

$r = 4$ or $r = -4$

11. $r = 6$

$\sqrt{x^2 + y^2} = 6$

$x^2 + y^2 = 36$

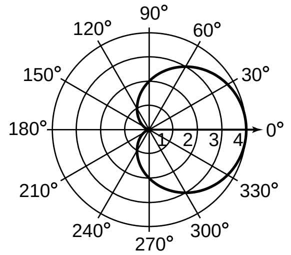
12. $r = -\sec \theta$

$\frac{r}{r} = \frac{-1}{r \cos \theta}$

$1 = \frac{-1}{x}$

$x = -1$

13a.



13b. No. The given point is on the negative x-axis, directly behind the microphone. The polar pattern indicates that the microphone does not pick up any sound from this direction.

Pages 572–573 Exercises

14. $r = \sqrt{2^2 + (-2)^2} = \sqrt{8}$ or $2\sqrt{2}$ $\quad \theta = \text{Arctan}\left(\frac{-2}{2}\right) = -\frac{\pi}{4}$

Add 2π to obtain $\theta = \frac{7\pi}{4}$.

$\left(2\sqrt{2}, \frac{7\pi}{4}\right)$

15. $r = \sqrt{0^2 + 1^2} = \sqrt{1}$ or 1

Since $x = 0$ when $y = 1$, $\theta = \frac{\pi}{2}$.

$\left(1, \frac{\pi}{2}\right)$

16. $r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4}$ or 2 $\quad \theta = \text{Arctan}\ \frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$\left(2, \frac{\pi}{3}\right)$

17. $r = \sqrt{\left(-\frac{1}{4}\right)^2 + \left(-\frac{\sqrt{3}}{4}\right)^2} = \sqrt{\frac{4}{16}} = \frac{2}{4}$ or $\frac{1}{2}$ $\quad \theta = \text{Arctan}\left(\dfrac{-\frac{\sqrt{3}}{4}}{-\frac{1}{4}}\right) = \text{Arctan}\left(\frac{\sqrt{3}}{1}\right)$ or $\frac{4\pi}{3}$

$\left(\frac{1}{2}, \frac{4\pi}{3}\right)$

18. $r = \sqrt{3^2 + 8^2} = \sqrt{73} \approx 8.54$ $\quad \theta = \text{Arctan}\left(\frac{8}{3}\right) \approx 1.21$

(8.54, 1.21)

19. $r = \sqrt{4^2 + (-7)^2} = \sqrt{65} \approx 8.06$ $\quad \theta = \text{Arctan}\left(\frac{-7}{4}\right) \approx -1.05$

Add 2π to obtain $\theta = 5.23$.

(8.06, 5.23)

20. $x = 3\cos\left(\frac{\pi}{2}\right) = 0$ $\quad y = 3\sin\left(\frac{\pi}{2}\right) = 3$

(0, 3)

21. $x = \frac{1}{2}\cos\left(\frac{3\pi}{4}\right) = \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{4}$ $\quad y = \frac{1}{2}\sin\left(\frac{3\pi}{4}\right) = \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}$

$\left(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$

22. $x = -1\cos\left(-\frac{\pi}{6}\right) = -1\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$ $\quad y = -1\sin\left(-\frac{\pi}{6}\right) = -1\left(-\frac{1}{2}\right) = \frac{1}{2}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

23. $x = -2\cos 270° = 0$ $\quad y = -2\sin 270° = 2$

(0, 2)

24. $x = 4\cos 210° = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$ $\quad y = 4\sin 210° = 4\left(-\frac{1}{2}\right) = -2$

$(-2\sqrt{3}, -2)$

25. $x = 14\cos 130° \approx -9.00$ $\quad y = 14\sin 130° \approx 10.72$

(−9.00, 10.72)

26.
$$\begin{aligned} x &= -7 \\ r\cos\theta &= -7 \\ r &= \frac{-7}{\cos\theta} \\ r &= -7\sec\theta \end{aligned}$$

27.
$$\begin{aligned} y &= 5 \\ r\sin\theta &= 5 \\ r &= \frac{5}{\sin\theta} \\ r &= 5\csc\theta \end{aligned}$$

28.
$$\begin{aligned} x^2 + y^2 &= 25 \\ (r\cos\theta)^2 + (r\sin\theta)^2 &= 25 \\ r^2(\cos^2\theta + \sin^2\theta) &= 25 \\ r^2 &= 25 \end{aligned}$$
$r = 5$ or $r = -5$

29.
$$\begin{aligned} x^2 + y^2 &= 2y \\ (r\cos\theta)^2 + (r\sin\theta)^2 &= 2r\sin\theta \\ r^2(\cos^2\theta + \sin^2\theta) &= 2r\sin\theta \\ r^2 &= 2r\sin\theta \\ r &= 2\sin\theta \end{aligned}$$

30.
$$\begin{aligned} x^2 - y^2 &= 1 \\ (r\cos\theta)^2 - (r\sin\theta)^2 &= 1 \\ r^2(\cos^2\theta - \sin^2\theta) &= 1 \\ r^2(\cos 2\theta) &= 1 \\ r^2 &= \frac{1}{\cos 2\theta} \\ r^2 &= \sec 2\theta \end{aligned}$$

31.
$$\begin{aligned} x^2 + (y-2)^2 &= 4 \\ x^2 + y^2 - 4y + 4 &= 4 \\ (r\cos\theta)^2 + (r\sin\theta)^2 - 4r\sin\theta &= 0 \\ r^2(\cos^2\theta + \sin^2\theta) - 4r\sin\theta &= 0 \\ r^2 - 4r\sin\theta &= 0 \\ r^2 &= 4r\sin\theta \\ r &= 4\sin\theta \end{aligned}$$

32.
$$\begin{aligned} r &= 2 \\ \sqrt{x^2 + y^2} &= 2 \\ x^2 + y^2 &= 4 \end{aligned}$$

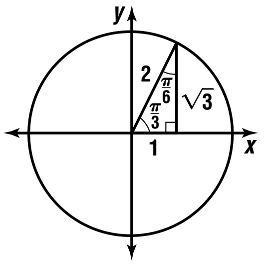
33.
$$\begin{aligned} r &= -3 \\ \sqrt{x^2 + y^2} &= -3 \\ x^2 + y^2 &= 9 \end{aligned}$$

34. $\theta = \frac{\pi}{3}$

$\text{Arctan}\,\frac{y}{x} = \frac{\pi}{3}$

$\frac{y}{x} = \frac{\sqrt{3}}{1}$

$y = \sqrt{3}x$



35. $r = 2 \csc \theta$

$\frac{r}{r} = \frac{2}{r \sin \theta}$

$1 = \frac{2}{y}$

$y = 2$

36. $r = 3 \cos \theta$

$r^2 = 3r \cos \theta$

$x^2 + y^2 = 3x$

37. $r^2 \sin 2\theta = 8$

$r^2 2 \sin \theta \cos \theta = 8$

$2r \sin \theta\, r \cos \theta = 8$

$2yx = 8$

$xy = 4$

38. $y = x$

$\frac{y}{x} = 1$

$\text{Arctan}\,\frac{y}{x} = \text{Arctan } 1$

$\theta = \frac{\pi}{4}$

39. $r = \sin \theta$

$r^2 = r \sin \theta$

$x^2 + y^2 = y$

40. $x = 325 \cos 70°$ ≈ 111.16 $\quad$ $y = 325 \sin 70°$ ≈ 305.40

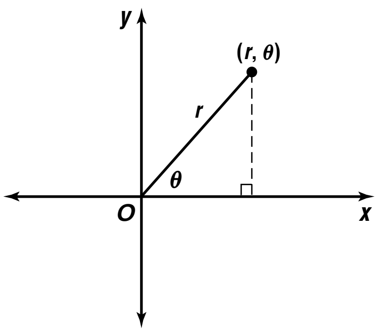
$(111.16, 305.40)$

41. $\dfrac{\frac{5\pi}{4}}{6} - \dfrac{\frac{\pi}{4}}{6} = \frac{5\pi}{24} - \frac{\pi}{24}$

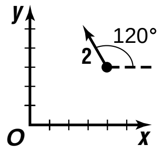
$= \frac{4\pi}{24}$

≈ 0.52 unit

42. Drop a perpendicular from the point with polar coordinates (r, θ) to the x-axis. r is the length of the hypotenuse in the resulting right triangle. x is the length of the side adjacent to angle θ, so $\cos \theta = \frac{x}{r}$. Solving for x gives $x = r \cos \theta$. y is the length of the side opposite angle θ, so $\sin \theta = \frac{y}{r}$. Solving for y gives $y = r \sin \theta$. (The figure is drawn for a point in the first quadrant, but the signs work out correctly regardless of where in the plane the point is located.)



43. horizontal distance:
$25(4 + 2 \cos 120°) = 75$ m east
vertical distance:
$25(3 + 2 \sin 120°) = 118.30$ m north



44a. $x = 4 \cos 20°$ ≈ 3.76 $\quad$ $y = 4 \sin 20°$ $= 1.37$

$\langle 3.76, 1.37 \rangle$

$x = 5 \cos 70°$ ≈ 1.71 $\quad$ $y = 5 \sin 70°$ ≈ 4.70

$\langle 1.71, 4.70 \rangle$

44b. $\langle 3.76, 1.37 \rangle + \langle 1.71, 4.70 \rangle$

$= \langle 3.76 + 1.71, 1.37 + 4.70 \rangle$

$= \langle 5.47, 6.07 \rangle$

44c. $5.47 = r \cos \theta$; $6.07 = r \sin \theta$

$\frac{6.07}{5.47} = \frac{r \sin \theta}{r \cos \theta}$

$\frac{6.07}{5.47} = \tan \theta$

$47.98 \approx \theta$; $47.98°$

$5.47 = r \cos 47.98°$

$\frac{5.47}{\cos 47.98°} = r$

$8.17 = r$

$8.17\angle 47.98°$

44d. $8.17 \sin (3.14t + 47.98°)$

45. $r = 2a \sin \theta + 2a \cos \theta$

$r^2 = 2ar \sin \theta + ar \cos \theta$

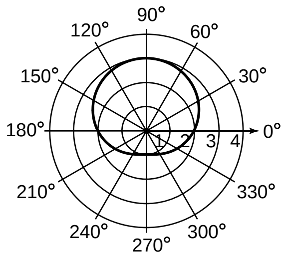
$x^2 + y^2 = 2ay + 2ax$

$x^2 - 2ax + y^2 - 2ay = 0$

$(x - a)^2 + (y - a)^2 = 2a^2$

The graph of the equation is the circle centered at (a, a) with radius $\sqrt{2}|a|$.

46.



47. Sample answer: $(-2, 405°)$, $(-2, 765°)$, $(2, 225°)$, $(2, 585°)$

$(r, \theta + 360k°)$

$\rightarrow (-2, 45° + 360(1)°) \rightarrow (-2, 405°)$

$\rightarrow (-2, 45° + 360(2)°) \rightarrow (-2, 765°)$

$(-r, \theta + (2k + 1)180°)$

$\rightarrow (2, 45° + (1)180°) \rightarrow (2, 225°)$

$\rightarrow (2, 45° + (3)180°) \rightarrow (2, 585°)$

48. $|\vec{r}|^2 = 50^2 + 425^2 = 2 \cdot 50 \cdot 425 \cos 30°$

$|\vec{r}| \approx 382.52$ mph

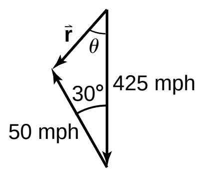
$\frac{50}{\sin \theta} = \frac{382.52}{\sin 30°}$

$50 \sin 30° = 382.52 \sin \theta$

$\frac{50 \sin 30°}{382.52} = \sin \theta$

$3°45' \approx \theta$

The direction is $3°45'$ west of south.



49. $\sin^2 A = \cos A - 1$

$1 - \cos^2 A = \cos A - 1$

$0 = \cos^2 A + \cos A - 2$

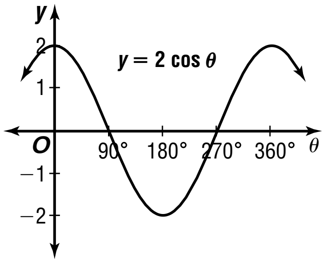
$0 = (\cos A + 2)(\cos A - 1)$

$\cos A + 2 = 0$ or $\cos A - 1 = 0$

$\cos A = -2$ $\quad$ $\cos A = 1$

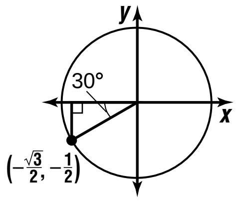
$A = 0°$

50.



51. The terminal side is in the third quadrant and the reference angle is $210 - 180$ or $30°$.

$\cos 210° = -\frac{\sqrt{3}}{2}$



52. Enter the x-values in L1 and the $f(x)$-values in L2 of your graphing calculator. Make a scatter plot. The data points are in the shape of parabola. Perform a quadratic regression.

$a \approx -0.07$, $b \approx 0.73$, $c \approx -1.36$

Sample answer:

$y = -0.07x^2 + 0.73x - 1.36$

53.
$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & -3 & 0 & -20 \\ & & 2 & 4 & 8 & 10 & 20 \\ \hline & 1 & 2 & 4 & 5 & 10 & 0 \end{array}$$

$x^4 + 2x^3 + 4x^2 + 5x + 10$

54. $m = \frac{625 - 145}{25 - 17}$ $\quad$ $(y - 145) = 60(x - 17)$

$= 60$ $\quad$ $y = 60x - 875$

55. $x > y$ and $y > z$, so $x > z$.

If $x > z$, then $0 < \frac{z}{x} < 1$.

The correct choice is C.

9-4 Polar Form of a Linear Equation

Pages 577–578 Check for Understanding

1. The polar equation of a line is $p = r \cos (\theta - \phi)$. r and θ are the variables. p is the length of the normal segment from the line to the origin and ϕ is the angle the normal makes with the positive x-axis.
2. For r to be equal to p, we must have $\cos (\theta - \phi) = 1$. The first positive value of θ for which this is true is $\theta = \phi$.
3. The graph of the equation $x = k$ is a vertical line. Since the line is vertical, the x-axis is the normal line through the origin. Therefore, $\phi = 0°$ or $\phi = 180°$, depending on whether k is positive or negative, respectively. The origin is $|k|$ units from the given vertical line, so $p = |k|$. The polar form of the given line is $k = r \cos (\theta - 0°)$ if k is positive or $-k = r \cos (\theta - 180°)$ if k is negative. Both equations simplify to $k = r \cos \theta$.
4. You can use the extra ordered pairs as a check on your work. If all the ordered pairs you plot are not collinear, then you have made a mistake.
5. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{3^2 + (-4)^2}$

   $= \pm 5$

   Since C is negative, use $+5$.

   $\frac{3}{5}x - \frac{4}{5}y - 2 = 0$

   $\cos \phi = \frac{3}{5}$, $\sin \phi = -\frac{4}{5}$, $p = 2$

   $\phi = \text{Arctan } -\frac{4}{3}$

   $\approx -53°$ or $307°$

   $p = r \cos (\theta - \phi)$

   $2 = r \cos (\theta - 307°)$
6. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{(-2)^2 + 4^2}$

   $= \pm 2\sqrt{5}$

   Since C is negative, use $+2\sqrt{5}$.

   $-\frac{2}{2\sqrt{5}} x + \frac{4}{2\sqrt{5}} y - \frac{9}{2\sqrt{5}} = 0$

   $\cos \phi = -\frac{\sqrt{5}}{5}$, $\sin \phi = \frac{2\sqrt{5}}{5}$, $p = \frac{9\sqrt{5}}{10}$

   $\phi = \text{Arctan}(-2)$

   $\approx -63°$

   Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.

   $\phi = 180° - 63°$ or $117°$

   $p = r \cos (\theta - \phi)$

   $\frac{9\sqrt{5}}{10} = r \cos (\theta - 117°)$
7. $3 = r \cos (\theta - 60°)$

   $0 = r \cos (\theta - 60°) - 3$

   $0 = r (\cos \theta \cos 60° + \sin \theta \sin 60°) - 3$

   $0 = \frac{1}{2}r \cos \theta + \frac{\sqrt{3}}{2} r \sin \theta - 3$

   $0 = \frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3$

   $0 = x + \sqrt{3}y - 6$ or

   $x + \sqrt{3}y - 6 = 0$
8. $r = 2 \sec \left(\theta + \frac{\pi}{4}\right)$

   $r \cos \left(\theta + \frac{\pi}{4}\right) = 2$

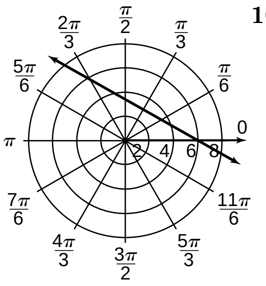
   $r\left(\cos \theta \cos \frac{\pi}{4} - \sin \theta \sin \frac{\pi}{4}\right) - 2 = 0$

   $\frac{\sqrt{2}}{2} r \cos \theta - \frac{\sqrt{2}}{2} r \sin \theta - 2 = 0$

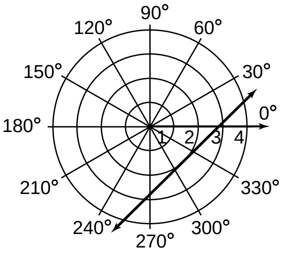
   $\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 2 = 0$

   $\sqrt{2}x - \sqrt{2}y - 4 = 0$

9.



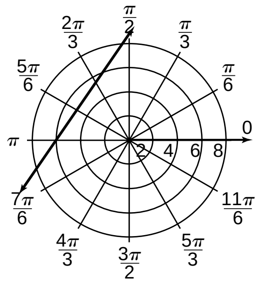
10.



11a. $p = r\cos(\theta - \phi) \to 5 = r\cos\left(\theta - \frac{5\pi}{6}\right)$

Since the shortest distance is along the normal, the answer is (p, ϕ) or $\left(5, \frac{5\pi}{6}\right)$.

11b.



Pages 578–579 Exercises

12. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{7^2 + (-24)^2}$
$= \pm 25$

Since C is positive, use -25.

$-\frac{7}{25}x + \frac{24}{25}y - 4 = 0$

$\cos\phi = -\frac{7}{25}, \sin\phi = \frac{24}{25}, p = 4$

$\phi = \text{Arctan } -\frac{24}{7}$

$\approx -74°$

Since $\cos\phi < 0$, but $\sin\phi > 0$, the normal lies in the second quadrant.

$\phi = 180° - 74°$ or $106°$

$p = r\cos(\theta - \phi)$

$4 = r\cos(\theta - 106°)$

13. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{21^2 + 20^2}$
$= \pm 29$

Since C is negative, use $+29$.

$\frac{21}{29}x + \frac{20}{29}y - \frac{87}{29} = 0$

$\cos\phi = \frac{21}{29}, \sin\phi = \frac{20}{29}, p = 3$

$\phi = \text{Arctan } \frac{20}{21}$

$\approx 44°$

$p = r\cos(\theta - \phi)$

$3 = r\cos(\theta - 44°)$

14. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{6^2 + (-8)^2}$
$= \pm 10$

Since C is negative, use $+10$.

$\frac{6}{10}x - \frac{8}{10}y - \frac{21}{10} = 0$

$\cos\phi = \frac{3}{5}, \sin\phi = -\frac{4}{5}, p = 2.1$

$\phi = \text{Arctan } -\frac{4}{3}$

$\approx -53°$

Since $\cos\phi > 0$, but $\sin\phi < 0$, the normal lies in the fourth quadrant.

$\phi = 360° - 53°$ or $307°$

$p = r\cos(\theta - \phi)$

$2.1 = r\cos(\theta - 307°)$

15. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{3^2 + 2^2}$
$= \pm\sqrt{13}$

Sinc C is negative, use $\pm\sqrt{13}$.

$\frac{3}{\sqrt{13}}x + \frac{2}{\sqrt{13}}y - \frac{5}{\sqrt{13}} = 0$

$\cos\phi = \frac{3\sqrt{13}}{13}, \sin\phi = \frac{2\sqrt{13}}{13}, p = \frac{5\sqrt{13}}{13}$

$\phi = \text{Arctan}\left(\frac{2}{3}\right)$

$\approx 34°$

$p = r\cos(\theta - \phi)$

$\frac{5\sqrt{13}}{13} = r\cos(\theta - 34°)$

16. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{4^2 + (-5)^2}$
$= \pm\sqrt{14}$

Since C is negative, use $\pm\sqrt{41}$.

$\frac{4}{\sqrt{41}}x - \frac{5}{\sqrt{41}}y - \frac{10}{\sqrt{41}} = 0$

$\cos\phi = \frac{4\sqrt{41}}{41}, \sin\phi = \frac{-5\sqrt{41}}{41}, p = \frac{10\sqrt{41}}{41}$

$\phi = \text{Arctan}\left(-\frac{5}{4}\right)$

$\approx -51°$

Since $\cos\phi > 0$, but $\sin\phi < 0$, the normal lies in the fourth quadrant.

$\phi = 360° - 51°$ or $309°$

$p = r\cos(\theta - \phi)$

$\frac{10\sqrt{41}}{41} = r\cos(\theta - 309°)$

17. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{(-1)^2 + 3^2}$
$= \pm\sqrt{10}$

Since C is negative, use $+\sqrt{10}$.

$-\frac{1}{\sqrt{10}}x + \frac{3}{\sqrt{10}}y - \frac{7}{\sqrt{10}} = 0$

$\cos\phi = -\frac{\sqrt{10}}{10} \sin\phi = \frac{3\sqrt{10}}{10}, p = \frac{7\sqrt{10}}{10}$

$\phi = \text{Arctan}(-3)$

$\approx -72°$

Since $\cos\phi < 0$, but $\sin\phi > 0$, the normal lies in the second quadrant.

$\phi = 180° - 72°$ or $108°$

$p = r\cos(\theta - \phi)$

$\frac{7\sqrt{10}}{10} = r\cos(\theta - 108°)$

18. $6 = r\cos(\theta - 120°)$

$0 = r(\cos\theta\cos 120° + \sin\theta\sin 120°) - 6$

$0 = -\frac{1}{2}r\cos\theta + \frac{\sqrt{3}}{2}r\sin\theta - 6$

$0 = -\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 6$

$0 = -x + \sqrt{3}y - 12$ or

$-x + \sqrt{3}y - 12 = 0$

19. $4 = r\cos\left(\theta + \frac{\pi}{4}\right)$

$0 = r\left(\cos\theta\cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4}\right) - 4$

$0 = \frac{\sqrt{2}}{2}r\cos\theta - \frac{\sqrt{2}}{2}r\sin\theta - 4$

$0 = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - 4$

$0 = \sqrt{2}x - \sqrt{2}y - 8$ or

$\sqrt{2}x - \sqrt{2}y - 8 = 0$

20. $2 = r\cos(\theta + \pi)$

$0 = r(\cos\theta\cos\pi - \sin\theta\sin\pi) - 2$

$0 = -r\cos\theta - 2$

$0 = -x - 2$

$x = -2$

21. $1 = r\cos(\theta - 330°)$

$0 = r(\cos\theta\cos 330° + \sin\theta\sin 330°) - 1$

$0 = \frac{\sqrt{3}}{2}r\cos\theta + \frac{1}{2}r\sin\theta - 1$

$0 = \frac{\sqrt{3}}{2}x - \frac{1}{2}y - 1$

$0 = \sqrt{3}x - y - 2$ or

$\sqrt{3}x - y - 2 = 0$

22. $r = 11\sec\left(\theta + \frac{7\pi}{6}\right)$

$r\cos\left(\theta + \frac{7\pi}{6}\right) = 11$

$r\left(\cos\theta\cos\frac{7\pi}{6} - \sin\theta\sin\frac{7\pi}{6}\right) - 11 = 0$

$-\frac{\sqrt{3}}{2}r\cos\theta + \frac{1}{2}r\sin\theta - 11 = 0$

$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - 11 = 0$

$-\sqrt{3}x + y - 22 = 0$

23. $r = 5\sec(\theta - 60°)$

$r\cos(\theta - 60°) = 5$

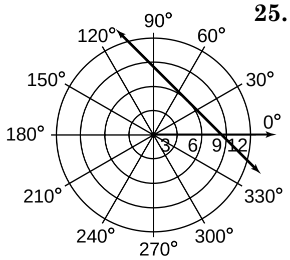
$r(\cos\theta\cos 60° + \sin\theta\sin 60°) - 5 = 0$

$\frac{1}{2}r\cos\theta + \frac{\sqrt{3}}{2}r\sin\theta - 5 = 0$

$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 5 = 0$

$x + \sqrt{3}y - 10 = 0$

24.



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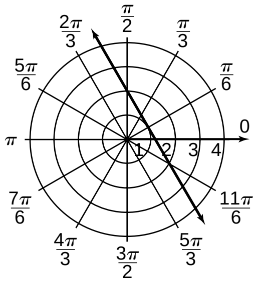

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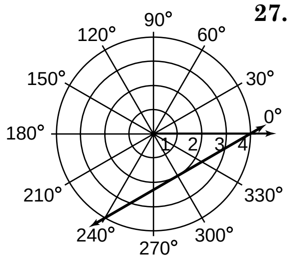

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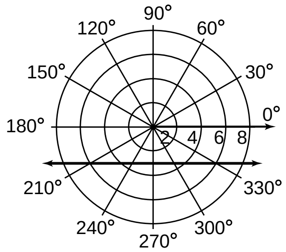

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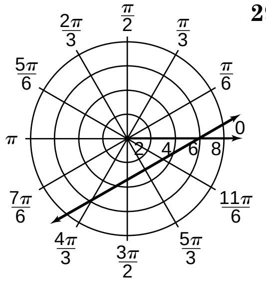

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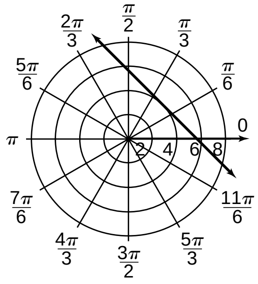


30. $m = \frac{4}{-6}$ or $-\frac{2}{3}$

$(y + 1) = -\frac{2}{3}(x - 4) \rightarrow 2x + 3y - 5 = 0$

$\pm\sqrt{A^2 + B^2} = \pm\sqrt{2^2 + 3^2}$

$= \pm\sqrt{13}$

Since C is negative, use $+\sqrt{13}$.

$\frac{2}{\sqrt{13}} + \frac{3}{\sqrt{13}} - \frac{5}{\sqrt{13}} = 0$

$\cos\phi = \frac{2\sqrt{13}}{13}$, $\sin\phi = \frac{3\sqrt{13}}{13}$, $p = \frac{5\sqrt{13}}{13}$

$\phi = \text{Arctan}\,\frac{3}{2}$

$\approx 56°$

$p = r\cos(\theta - \phi)$

$\frac{5\sqrt{13}}{13} = r\cos(\theta - 56°)$

31. $p = r\cos(\theta - \phi)$

$\rightarrow p = 3\cos\left(\frac{\pi}{4} - \phi\right)$

$\rightarrow p = 2\cos\left(\frac{7\pi}{6} - \phi\right)$

Use a graphing calculator and the INTERJECT feature to find solutions to the system at (2.25, 0.31) and (5.39, −0.31). Since p, the length of the normal, must be positive, use $\phi = 2.25$ and $p = 0.31$.

$0.31 = r\cos(\theta - 2.25)$

32a. $p = r\cos(\theta - \phi) \rightarrow 6 = r\cos(\theta - 16°)$
Since the shortest distance is along the normal, the closest the fly came was p or 6 cm.

32b. (p, ϕ) or $(6, 15°)$

33. Since both normal segments have length 2, p must be 2 in both equations. Since the two lines intersect at right angles, their normals also intersect at right angles. This can be achieved by having the two ϕ-values differ by 90°. To make sure neither line is vertical, neither ϕ-value should be a multiple of 90°. Therefore, a sample answer is $2 = r\cos(\theta - 45°)$ and $2 = r\cos(\theta - 135°)$.

34. $m = 0$

$(y - 4) = 0(x - 5) \rightarrow y - 4 = 0$

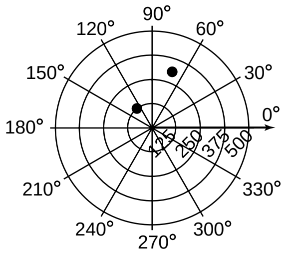
$\cos\phi = 0$, $\sin\phi = 1$, $p = 4$

Since $\cos\phi = 0$ when $\sin\phi = 1$, $\phi = 90°$.

$p = r\cos(\theta - \phi)$

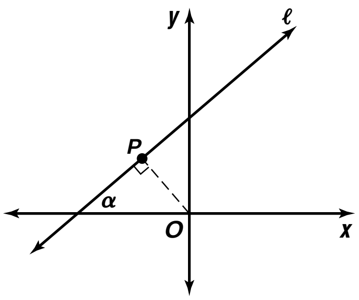
$4 = r\cos(\theta - 90°)$

35a.



35b. $p = r\cos(\theta - \phi)$
$\rightarrow p = 125\cos(130 - \phi)$
$\rightarrow p = 300\cos(70 - \phi)$
Use a graphing calculator and the INTERSECT feature to find the solutions to the system at $(-45, -124.43)$ and $(135, 124.43)$. Sinc p, the length of the normal, must be positive, use $\phi = 135°$ and $p = 124.43$.
$124.43 = r\cos(\theta - 135°)$

36. $k = r\sin(\theta - \alpha)$
$k = r[\sin\theta\cos\alpha - \cos\theta\sin\alpha]$
$k = r\sin\theta\cos\alpha - r\cos\theta\sin\alpha$
$k = y\cos\alpha - x\sin\alpha$
This is the equation of a line in rectangular coordinates. Solving the last equation for y yields $y = (\tan\alpha)x + \frac{k}{\cos\alpha}$. The slope of the line shows that α is the angle the line makes with the x-axis. To find the length of the normal segment in the figure, observe that the complementary angle to α in the right triangle is $90° - \alpha$, so the θ-coordinate of P in polar coordinates is $180° - (90° - \alpha) = \alpha + 90°$. Substitute into the original polar equation to find the r-coordinate of P:
$k = r\sin(\alpha + 90° - \alpha)$
$k = r\sin 90°$
$k = r$
Therefore, k is the length of the normal segment.



37. $p = r\cos(\theta - \phi)$
$\rightarrow p = 40\cos(0° - \phi)$
$\rightarrow p = 40\cos(72° - \phi)$
Use a graphing calculator and the INTERSECT feature to find the solutions of the system at $(-144, -32.36)$ and $(36, 32.36)$. Since p, the length of the normal, must be positive, use $\phi = 36°$ and $p = 32.36$.
$32.36 = r\cos(\theta - 36°)$

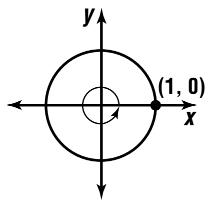
38. $r = 6$
$\sqrt{x^2 + y^2} = 6$
$x^2 + y^2 = 36$

39. The graph of a polar equation of the form $r = a\sin n\theta$ is a rose.

40. $x - 3y = 6$
$y = \frac{-x + 6}{-3}$
$y = \frac{1}{3}x - 2$
$x = t,\ y = \frac{1}{3}t - 2$

41. $A = \frac{N}{360}\left(\pi r^2\right)$
$= \frac{65}{360}\left(\pi 6^2\right)$
$\approx 20.42 \text{ ft}^2$

42. Since 360° lies on the x-axis of the unit circle at (1, 0), $\sin 360° = y$ or 0.

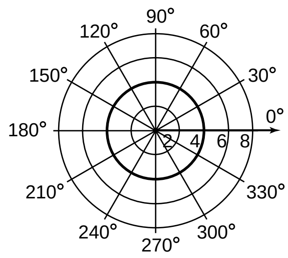


43. $2x^3 + 5x^2 - 12x = 0$
$x(2x^2 + 5x - 12) = 0$
$x(2x - 3)(x + 4) = 0$
$x = 0$ or $x = \frac{3}{2}$ or $x = -4$

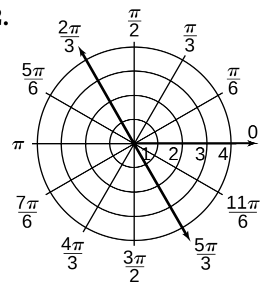
44. $c^2 - d^2 = 48$
$(c + d)(c - d) = 48$
$12(c - d) = 48$
$c - d = 4$

Page 579 Mid-Chapter Quiz

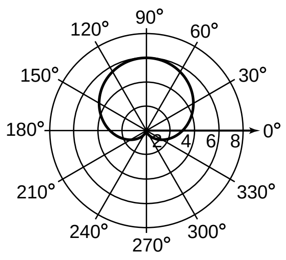
1.



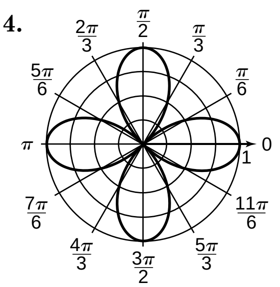
2.



3.



4.



5. $r = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-\sqrt{2}}{-\sqrt{2}}\right)$
$= \sqrt{4}$ or 2 $\qquad = \frac{\pi}{4}$
Since $(-\sqrt{2}, -\sqrt{2})$ is in the third quadrant, $\theta = \pi + \frac{\pi}{4}$ or $\frac{5\pi}{4}$.
$\left(2, \frac{5\pi}{4}\right)$

6. $r = \sqrt{0^2 + (-4)^2}$
$= \sqrt{16}$ or 4
Since $x = 0$ when $y = -4$, $\theta = \frac{3\pi}{2}$.
$\left(4, \frac{3\pi}{2}\right)$

7. $x^2 + y^2 = 36$
$\sqrt{x^2 + y^2} = \pm\sqrt{36}$
$r = 6$ or $r = -6$

8. $r = 2 \csc \theta$
$r \sin \theta = 2$
$y = 2$

9. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{5^2 + (-12)^2}$
$= \pm 13$
Since C is positive, use -13.
$-\frac{5}{13}x + \frac{12}{13}y - \frac{3}{13} = 0$
$\cos \phi = -\frac{5}{13}$, $\sin \phi = \frac{12}{13}$, $p = \frac{3}{13}$
$\phi = \text{Arctan}\left(-\frac{12}{5}\right)$
$\approx -67°$
Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.
$\phi = 180° - 67°$ or $113°$
$p = r \cos(-\phi)$
$\frac{3}{13} = r \cos(\theta - 113°)$

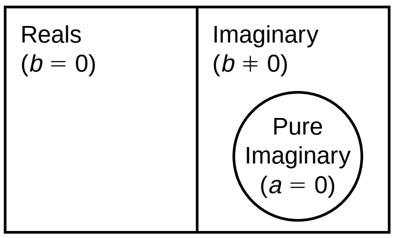
10. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{(-2)^2 + (-6)^2}$
$= \pm 2\sqrt{10}$
Since C is negative, use $+2\sqrt{10}$.
$-\frac{2}{2\sqrt{10}x} - \frac{6}{2\sqrt{10}}y - \frac{2}{2\sqrt{10}} = 0$
$\cos \phi = -\frac{\sqrt{10}}{10}$, $\sin \phi = -\frac{3\sqrt{10}}{10}$, $p = \frac{\sqrt{10}}{10}$
$\phi = \text{Arctan}\left(\frac{-3}{-1}\right)$
$\approx 72°$
Since $\cos \phi < 0$ and $\sin \phi < 0$, the normal lies in the third quadrant.
$\phi = 180° + 72°$ or $252°$
$p = r \cos(\theta - \phi)$
$\frac{\sqrt{10}}{10} = r \cos(\theta - 252°)$

9-5 Simplifying Complex Numbers

Page 583 Check for Understanding

1. Find the (positive) remainder when the exponent is divided by 4. If the remainder is 0, the answer is 1; if the remainder is 1, the answer is $\boldsymbol{i}$; if the remainder is 2, the answer is -1; and if the remainder is 3, the answer is $-\boldsymbol{i}$.

2. **Complex Numbers ($a + bi$)**



3. When you multiply the denominators, you will be multiplying a complex number and its conjugate. This makes the denominator of the product a real number, so you can then write the answer in the form $a + b\boldsymbol{i}$.

4. Sample answer: $x^2 + 1 = 0$
$(x - \boldsymbol{i})(x + \boldsymbol{i}) = 0$, where the solutions are $x = \pm\boldsymbol{i}$.
$x^2 + x\boldsymbol{i} - x\boldsymbol{i} - \boldsymbol{i}^2 = 0$
$x^2 - (-1) = 0$
$x^2 + 1 = 0$

5. $\boldsymbol{i}^{-6} = (\boldsymbol{i}^4)^{-2} \cdot \boldsymbol{i}^2$
$= 1^{-2} \cdot (-1)$
$= -1$

6. $\boldsymbol{i}^{10} + \boldsymbol{i}^2 = (\boldsymbol{i}^4)^2 \cdot \boldsymbol{i}^2 + \boldsymbol{i}^2$
$= (1)^2\boldsymbol{i}^2 + \boldsymbol{i}^2$
$= -1 + (-1)$ or -2

7. $(2 + 3\boldsymbol{i}) + (-6 + \boldsymbol{i}) = (2 + (-6)) + (3\boldsymbol{i} + \boldsymbol{i})$
$= -4 + 4\boldsymbol{i}$

8. $(2.3 + 4.1\boldsymbol{i}) - (-1.2 - 6.3\boldsymbol{i})$
$= (2.3 - (-1.2)) + (4.1\boldsymbol{i} - (-6.3\boldsymbol{i}))$
$= 3.5 + 10.4\boldsymbol{i}$

9. $(2 + 4\boldsymbol{i}) + (-1 + 5\boldsymbol{i}) = (2 + (-1)) + (4\boldsymbol{i} + 5\boldsymbol{i})$
$= 1 + 9\boldsymbol{i}$

10. $(-2 - \boldsymbol{i})^2 = (-2 - \boldsymbol{i})(-2 - \boldsymbol{i})$
$= 4 + 4\boldsymbol{i} + \boldsymbol{i}^2$
$= 3 + 4\boldsymbol{i}$

11. $\frac{\boldsymbol{i}}{1 + 2\boldsymbol{i}} = \frac{\boldsymbol{i}}{1 + 2\boldsymbol{i}} \cdot \frac{1 - 2\boldsymbol{i}}{1 - 2\boldsymbol{i}}$
$= \frac{\boldsymbol{i} - 2\boldsymbol{i}^2}{1 - 4\boldsymbol{i}^2}$
$= \frac{\boldsymbol{i} + 2}{5}$
$= \frac{2}{5} + \frac{1}{5}\boldsymbol{i}$

12. $(2.5 + 3.1\boldsymbol{i}) + (-6.2 + 4.3\boldsymbol{i})$
$= (2.5 + (-6.2)) + (3.1\boldsymbol{i} + 4.3\boldsymbol{i})$
$= -3.7 + 7.4\boldsymbol{i}$ N

Pages 583–585 Exercises

13. $\boldsymbol{i}^6 = \boldsymbol{i}^4 \cdot \boldsymbol{i}^2$
$= 1 \cdot -1$
$= -1$

14. $\boldsymbol{i}^{19} = (\boldsymbol{i}^4)^4 \cdot \boldsymbol{i}^3$
$= 1^4 \cdot -\boldsymbol{i}$
$= -\boldsymbol{i}$

15. $\boldsymbol{i}^{1776} = (\boldsymbol{i}^4)^{444}$
$= 1^{444}$
$= 1$

16. $\boldsymbol{i}^9 + \boldsymbol{i}^{-5} = (\boldsymbol{i}^4)^2 \cdot \boldsymbol{i} + (\boldsymbol{i}^4)^{-2} \cdot \boldsymbol{i}^3$
$= 1^2 \cdot \boldsymbol{i} + 1^{-2} \cdot -\boldsymbol{i}$
$= \boldsymbol{i} + (-\boldsymbol{i})$ or 0

17. $(3 + 2\boldsymbol{i}) + (-4 + 6\boldsymbol{i}) = (3 + (-4)) + (2\boldsymbol{i} + 6\boldsymbol{i})$
$= -1 + 8\boldsymbol{i}$

18. $(7 - 4\boldsymbol{i}) + (2 - 3\boldsymbol{i}) = (7 + 2) + (-4\boldsymbol{i} - 3\boldsymbol{i})$
$= 9 - 7\boldsymbol{i}$

19. $\left(\frac{1}{2} + \boldsymbol{i}\right) - (2 - \boldsymbol{i}) = \left(\frac{1}{2} + (-2)\right) + (\boldsymbol{i} - (-\boldsymbol{i}))$
$= -\frac{3}{2} + 2\boldsymbol{i}$

20. $(-3 - \boldsymbol{i}) - (4 - 5\boldsymbol{i}) = (-3 + (-4)) + (-\boldsymbol{i} - (-5\boldsymbol{i}))$
$= -7 + 4\boldsymbol{i}$

21. $(2 + \boldsymbol{i})(4 + 3\boldsymbol{i}) = 8 + 10\boldsymbol{i} + 3\boldsymbol{i}^2$
$= 5 + 10\boldsymbol{i}$

22. $(1 + 4\boldsymbol{i})^2 = (1 + 4\boldsymbol{i})(1 + 4\boldsymbol{i})$
$= 1 + 8\boldsymbol{i} + 16\boldsymbol{i}^2$
$= -15 + 8\boldsymbol{i}$

23. $(1 + \sqrt{7}i)(-2 - \sqrt{5}i)$
$= -2 - \sqrt{5}i - 2\sqrt{7}i - \sqrt{35}i^2$
$= (-2 + \sqrt{35}) + (-2\sqrt{7} - \sqrt{5})i$

24. $(2 + \sqrt{-3})(-1 + \sqrt{-12}) = (2 + \sqrt{3}i)(-1 + \sqrt{12}i)$
$= -2 + 2\sqrt{12}i - \sqrt{3}i + \sqrt{36}i^2$
$= -2 + 4\sqrt{3}i - \sqrt{3}i - 6$
$= -8 + 3\sqrt{3}i$

25. $\frac{2+i}{1+2i} = \frac{2+i}{1+2i} \cdot \frac{1-2i}{1-2i}$
$= \frac{2 - 3i - 2i^2}{1 - 4i^2}$
$= \frac{4 - 3i}{5}$
$= \frac{4}{5} - \frac{3}{5}i$

26. $\frac{3-2i}{-4-i} = \frac{3-2i}{-4-i} \cdot \frac{-4+i}{-4+i}$
$= \frac{-12 + 11i - 2i^2}{16 - i^2}$
$= \frac{-10 + 11i}{17}$
$= -\frac{10}{17} + \frac{11}{17}i$

27. $\frac{5-i}{5+i} = \frac{5-i}{5+i} \cdot \frac{5-i}{5-i}$
$= \frac{25 - 10i + i^2}{25 - i^2}$
$= \frac{24 - 10i}{26}$
$= \frac{12}{13} - \frac{5}{13}i$

28. $(x - i)(x + i) = 0$
$x^2 - i^2 = 0$
$x^2 + 1 = 0$

29.
$(x - (2 + i))(x - (2 - i)) = 0$
$(x - 2 - i)(x - 2 + i) = 0$
$x^2 - 2x + xi - 2x + 4 - 2i - xi + 2i - i^2 = 0$
$x^2 - 4x + 4 + 1 = 0$
$x^2 - 4x + 5 = 0$

30. $(2 - i)(3 + 2i)(1 - 4i) = (6 + i - 2i^2)(1 - 4i)$
$= (8 + i)(1 - 4i)$
$= 8 - 31i - 4i^2$
$= 12 - 31i$

31. $(-1 - 3i)(2 + 2i)(1 - 2i) = (-2 - 8i - 6i^2)(1 - 2i)$
$= (4 - 8i)(1 - 2i)$
$= 4 - 16i + 16i^2$
$= -12 - 16i$

32. $\frac{\frac{1}{2} + \sqrt{3}i}{1 - \sqrt{2}i} = \frac{\frac{1}{2} + \sqrt{3}i}{1 - \sqrt{2}i} \cdot \frac{1 + \sqrt{2}i}{1 + \sqrt{2}i}$
$= \frac{\frac{1}{2} + \frac{\sqrt{2}}{2}i + 3i + 6i^2}{1 - 2i^2}$
$= \frac{\left(\frac{1}{2} - \sqrt{6}\right) + \left(\frac{\sqrt{2}}{2} + \sqrt{3}\right)i}{3}$
$= \left(\frac{1}{6} - \frac{\sqrt{6}}{3}\right) + \left(\frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{6}\right)i$

33. $\frac{2 - \sqrt{2}i}{3 + \sqrt{6}i} = \frac{2 - \sqrt{2}i}{3 + \sqrt{6}i} \cdot \frac{3 - \sqrt{6}i}{3 - \sqrt{6}i}$
$= \frac{6 - 2\sqrt{6}i - 3\sqrt{2}i + \sqrt{12}i^2}{9 - 6i^2}$
$= \frac{(6 - 2\sqrt{3}) + (-2\sqrt{6} - 3\sqrt{2})i}{15}$
$= \left(\frac{2}{5} - \frac{2\sqrt{3}}{15}\right) + \left(-\frac{\sqrt{2}}{5} - \frac{2\sqrt{6}}{15}\right)i$

34. $\frac{3+i}{(2+i)^2} = \frac{3+i}{(2+i)(2+i)}$
$= \frac{3+i}{4 + 4i + i^2}$
$= \frac{3+i}{3+4i}$
$= \frac{3+i}{3+4i} \cdot \frac{3-4i}{3-4i}$
$= \frac{9 - 9i - 4i^2}{9 - 16i^2}$
$= \frac{13 - 9i}{25}$
$= \frac{13}{25} - \frac{9}{25}i$

35. $\frac{(1+i)^2}{(-3+2i)^2} = \frac{(1+i)(1+i)}{(-3+2i)(-3+2i)}$
$= \frac{1 + 2i + i^2}{9 - 12i + 4i^2}$
$= \frac{2i}{5 - 12i}$
$= \frac{2i}{5 - 12i} \cdot \frac{5 + 12i}{5 + 12i}$
$= \frac{10i + 24i^2}{25 - 144i^2}$
$= \frac{-24 + 10i}{169}$
$= -\frac{24}{169} + \frac{10}{169}i$

36a. $Z = R + (X_L - X_C)j$
$\rightarrow Z = 10 + (1 - 2)j \rightarrow Z = 10 - j$ ohms
$\rightarrow Z = 3 + (1 - 1)j \rightarrow Z = 3 + 0j$ ohms

36b. $(10 - j) + (3 + 0j) = (10 + 3) + (-1j + 0j)$
$= 13 - j$ ohms

36c. $S = \frac{1}{Z} \rightarrow S = \frac{1}{6 + 3j}$
$= \frac{1}{6 + 3j} \cdot \frac{6 - 3j}{6 - 3j}$
$= \frac{6 - 3j}{36 - 9j^2}$
$= \frac{6 - 3j}{45}$
$\approx 0.13 - 0.07j$ siemens

37a. $x = \frac{-8i \pm \sqrt{(8i)^2 - 4(1)(-25)}}{2(1)}$
$= \frac{-8i \pm \sqrt{36}}{2}$
$= \pm 3 - 4i$

37b. No

37c. The solutions need not be complex conjugates because the coefficients in the equation are not all real.

37d. $(3 - 4i)^2 + 8i(3 - 4i) - 25 \stackrel{?}{=} 0$
$-7 - 24i + 24i + 32 - 25 \stackrel{?}{=} 0$
$0 = 0$
$(-3 - 4i)^2 + 8i(-3 - 4i) - 25 \stackrel{?}{=} 0$
$-7 + 24i - 24i + 32 - 25 \stackrel{?}{=} 0$
$0 = 0$

38. $f(x + yi) = (x + yi)^2$
$= x^2 + 2xyi - y^2$
$= (x^2 - y^2) + 2xyi$

39a. $z_0 = 2 - i$
$z_1 = i(2 - i) + i^2$ or $1 + 2i$
$z_2 = i(2i + 1) = 2i^2 + i$ or $-2 + i$
$z_3 = i(-2 + i) = -2i + i^2$ or $-1 - 2i$
$z_4 = i(-1 - 2i) = -i - 2i^2$ or $2 - i$
$z_5 = i(2 - i) = 2i - i^2$ or $1 + 2i$

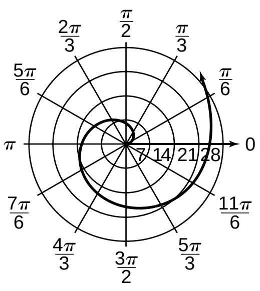
39b. $z_0 = 1 + 0\boldsymbol{i}$
$z_1 = (0.5 - 0.866\boldsymbol{i})(1 + 0\boldsymbol{i}) = 0.5 - 0.866\boldsymbol{i}$
$z_2 = (0.5 - 0.866\boldsymbol{i})(0.5 - 0.866\boldsymbol{i})$
$= 0.25 - 0.866\boldsymbol{i} - 0.75$
$= -0.500 - 0.866\boldsymbol{i}$
$z_3 = (0.5 - 0.866\boldsymbol{i})(-0.500 - 0.866\boldsymbol{i})$
$= -0.250 - 0.750$
$= -1.000 - 0.000\boldsymbol{i}$
$z_4 = (0.5 - 0.866\boldsymbol{i})(-1.000) = -0.500 + 0.866\boldsymbol{i}$
$z_5 = (0.5 - 0.866\boldsymbol{i})(-0.500 + 0.866\boldsymbol{i})$
$= -0.250 + 0.866\boldsymbol{i} + 0.75$
$= 0.500 + 0.866\boldsymbol{i}$

40. $(1 + 2\boldsymbol{i})^{-3} = \frac{1}{(1 + 2\boldsymbol{i})^3}$
$= \frac{1}{(-3 + 4\boldsymbol{i})(1 + 2\boldsymbol{i})}$
$= \frac{1}{-11 - 2\boldsymbol{i}}$
$= \frac{1}{-11 - 2\boldsymbol{i}} \cdot \frac{-11 + 2\boldsymbol{i}}{-11 + 2\boldsymbol{i}}$
$= \frac{-11 + 2\boldsymbol{i}}{125}$
$= -\frac{11}{125} + \frac{2}{125}\boldsymbol{i}$

41. $c_1(\cos 2t + \boldsymbol{i} \sin 2t) + c_2(\cos 2t - \boldsymbol{i} \sin 2t)$
$= c_1 \cos 2t + c_1\boldsymbol{i} \sin 2t + c_2 \cos 2t - c_2\boldsymbol{i} \sin 2t$
$= (c_1 + c_2)(\cos 2t) + (c_1 - c_2)(\boldsymbol{i} \sin 2t)$
$= (c_1 + c_2)(\cos 2t)$ only if $c_1 = c_2$

42. $\pm\sqrt{A^2 + b^2} = \pm\sqrt{6^2 + (-2)^2}$
$= \pm 2\sqrt{10}$
Since C is positive, use $-2\sqrt{10}$.
$-\frac{6}{2\sqrt{10}}x + \frac{2}{2\sqrt{10}}y - \frac{3}{2\sqrt{10}} = 0$
$\cos \phi = -\frac{3\sqrt{10}}{10}$, $\sin \phi = \frac{\sqrt{10}}{10}$, $p = \frac{3\sqrt{10}}{20}$
$\phi = \text{Arctan}\left(-\frac{1}{3}\right)$
$\approx -18°$
Since $\cos \phi < 0$, but $\sin \phi > 0$, the normal lies in the second quadrant.
$\phi = 180° - 18°$ or $162°$
$p = r \cos (\theta - \phi)$
$\frac{3\sqrt{10}}{20} = r \cos (\theta - 162°)$

43.



44. $\langle x - (-3), y - 6\rangle = t\langle 1, -4\rangle$
$\langle x + 3, y - 6\rangle = t\langle 1, -4\rangle$

45. $\vec{\boldsymbol{u}} = \frac{1}{4}\langle -8, 6, 4\rangle - 2\langle 2, -6, 3\rangle$
$= \left\langle -2, \frac{3}{2}, 1\right\rangle - \langle 4, -12, 6\rangle$
$= \left\langle -6, \frac{27}{2}, -5\right\rangle$

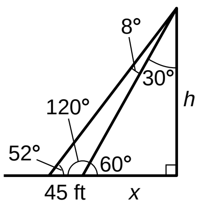
46. $\tan \alpha = \frac{4}{3}$
$\tan^2 \alpha + 1 = \sec^2 \alpha$
$\left(\frac{4}{3}\right)^2 + 1 = \sec^2 \alpha$
$\frac{25}{9} = \sec^2 \alpha$
$\frac{9}{25} = \cos^2 \alpha$
$\frac{3}{5} = \cos \alpha$
$\sin^2 \alpha + \cos^2 \alpha = 1$
$\sin^2 \alpha + \left(\frac{3}{5}\right)^2 = 1$
$\sin^2 \alpha = \frac{16}{25}$
$\sin \alpha = \frac{4}{5}$

$\cot B = \frac{5}{12}$
$1 + \cot^2 B = \csc^2 B$
$1 + \left(\frac{5}{12}\right)^2 = \csc^2 B$
$\frac{169}{144} = \csc^2 B$
$\frac{144}{169} = \sin^2 B$
$\frac{12}{13} = \sin B$
$\sin^2 B + \cos^2 B = 1$
$\left(\frac{12}{13}\right)^2 + \cos^2 B = 1$
$\cos^2 B = \frac{25}{169}$
$\cos B = \frac{5}{13}$

$\cos (\alpha + B) = \cos \alpha \cos B - \sin \alpha \sin B$
$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$
$= -\frac{33}{65}$

47. amplitude $= \frac{1}{2}(7)$ or 3.5
period $= \frac{2\pi}{12}$ or $\frac{\pi}{6}$
$y = 3.5 \cos\left(\frac{\pi}{6}t\right)$

48. $h = x\sqrt{3}$
$\tan 52° = \frac{x\sqrt{3}}{x + 45}$
$x \tan 52° + 45 \tan 52° = x\sqrt{3}$
$x \tan 52° - x\sqrt{3} = -45 \tan 52°$
$x = \frac{-45 \tan 52°}{\tan 52° - \sqrt{3}}$
$x \approx 127.40$
$h = x\sqrt{3} = 127.40(\sqrt{3}) \approx 221$ ft



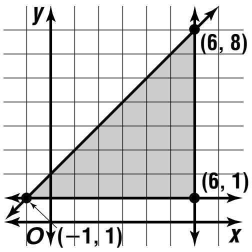
49. Enter the x-values in L1 and the $f(x)$-values in L2 of your graphing calculator. Make a scatter plot. The data points are in the shape of a parabola, so a quadratic function would best model the set of data.

50. Let d = depth of the original pool.
The second pool's width $= 5d + 4$, the length $= 10d + 6$, and the depth $= d + 2$.
$(5d + 4)(10d + 6)(d + 2) = 3420$
$(50d^2 + 70d + 24)(d + 2) = 3420$
$50d^3 + 100d^2 + 70d^2 + 140d + 24d + 48 = 3420$
$50d^3 + 170d^2 + 164d - 3372 = 0$
$25d^3 + 85d^2 + 82d - 1686 = 0$
Use a graphing calculator to find the solution $d = 3$.
The dimensions of the original pool are 15 ft by 30 ft by 3 ft.

51. $80 = k(5)(8)$
$2 = k$
$y = 2(16)(2)$
$= 64$

52.
$$\begin{aligned} y &= 7 - x^2 \\ x &= 7 - y^2 \\ x - 7 &= -y^2 \\ -x + 7 &= y^2 \\ \pm\sqrt{-x + 7} &= y \\ f^{-1}(x) &= \pm\sqrt{7 - x} \end{aligned}$$

53.



$f(x, y) = -2x + y$
$f(-1, 1) = -2(-1) + 1$ or 3
$f(6, 1) = -2(6) + 1$ or -11
$f(6, 8) = -2(6) + 8$ or -4
The maximum value is 3 and the minimum value is -11.

54.
$$\begin{array}{r} x + 2y - 7z = 14 \\ \underline{-x - 3y + 5z = -21} \\ -y - 2z = -7 \end{array}$$

$$\begin{array}{rcr} -x - 3y + 5z = -21 & \rightarrow & -5x - 15y + 25z = -105 \\ 5x - y + 2z = -7 & & \underline{5x - y + 2z = -7} \\ & & -16y + 27z = -112 \\ -y - 2z = -7 & \rightarrow & 16y + 32z = 112 \\ -16y + 27z = -112 & & \underline{-16y + 27z = -112} \\ & & 59z = 0 \\ & & z = 0 \end{array}$$

$-y - 2(0) = -7 \quad \rightarrow \quad y = 7$
$x + 2(7) - 7(0) = 14 \quad \rightarrow \quad x = 0$
$(0, 7, 0)$

55. Since $BC = BD$, $m\angle BDC = m\angle DCB = x$
$m\angle DBC = 180 - 120$ or 60.
$$\begin{aligned} x + x + 60 &= 180 \\ 2x &= 120 \\ x &= 60 \end{aligned}$$
$x + 40 = 60 + 40$ or 100
The correct choice is A.

9-6 The Complex and Polar Form of Complex Numbers

Pages 589–590 Check for Understanding

1. To find the absolute value of $a + b\boldsymbol{i}$, square a and b, add the squares, then take the square root of the sum.

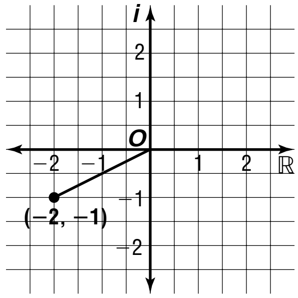
2. $\boldsymbol{i} = 0 + \boldsymbol{i}$; $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$
$\boldsymbol{i} = \cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}$

3. Sample answer: $z_1 = \boldsymbol{i}$, $z_2 = -\boldsymbol{i}$
$$\begin{aligned} |z_1 + z_2| &= |z_1| + |z_2| \\ |\boldsymbol{i} + (-\boldsymbol{i})| &\stackrel{?}{=} |\boldsymbol{i}| + |-\boldsymbol{i}| \\ |0| &\stackrel{?}{=} \boldsymbol{i} + \boldsymbol{i} \\ 0 &\neq 2\boldsymbol{i} \end{aligned}$$

4. The conjugate of $a + b\boldsymbol{i}$ is $a - b\boldsymbol{i}$.
$\sqrt{(a + b\boldsymbol{i})(a - b\boldsymbol{i})} = \sqrt{a^2 + b^2}$, so the friend's method gives the same answer.
Sample answer: The absolute value of $2 + 3\boldsymbol{i}$ is $\sqrt{2^2 + 3^2} = \sqrt{13}$. Using the friend's method, the absolute value is $\sqrt{(2 + 3\boldsymbol{i})(2 - 3\boldsymbol{i})} = \sqrt{4 + 9} = \sqrt{13}$.

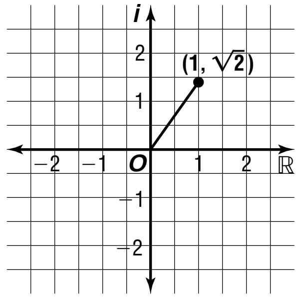
5. $2x + y + (x + y)\boldsymbol{i} = 5 + 4\boldsymbol{i}$
$$\begin{array}{ll} 2x + y = 5 & x + y = 4 \\ 2x + (-x + 4) = 5 & y = -x + 4 \\ x = 1 & \\ y = -(1) + 4 \text{ or } 3 & \end{array}$$

6.



$|z| = \sqrt{(-2^2 + (-1)^2}$
$= \sqrt{5}$

7.



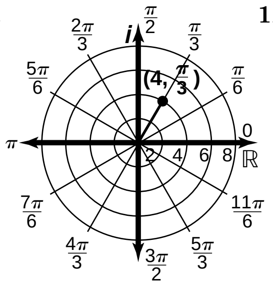
$|z| = \sqrt{1^2 + (\sqrt{2})^2}$
$= \sqrt{3}$

8. $r = \sqrt{2^2 + (-2)^2}$
$= \sqrt{8}$ or $2\sqrt{2}$
$\theta = \text{Arctan}\left(\frac{-2}{2}\right) + 2\pi$
$= \frac{7\pi}{4}$
θ is in the fourth quadrant.
$2 - 2\boldsymbol{i} = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + \boldsymbol{i}\sin\frac{7\pi}{4}\right)$

9. $r = \sqrt{4^2 + 5^2}$
$= \sqrt{41}$
$\theta = \text{Arctan}\left(\frac{5}{4}\right)$
$= 0.90$
$4 + 5\boldsymbol{i} = \sqrt{41}(\cos 0.90 + \boldsymbol{i}\sin 0.90)$

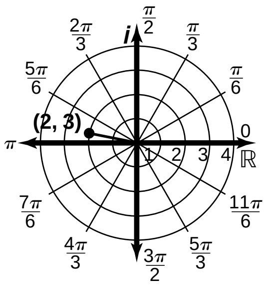
10. $r = \sqrt{(-2)^2 + 0^2}$
$= \sqrt{4}$ or 2
$\theta = \text{Arctan}\,\frac{0}{-2} + \pi$
$= \pi$
θ is on the x-axis at -2.
$-2 = 2(\cos\pi + \boldsymbol{i}\sin\pi)$

11.



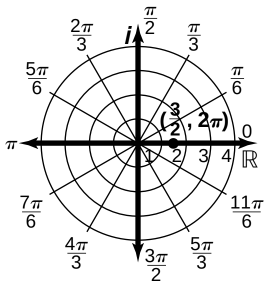
$4\left(\cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3}\right)$
$= 4\left(\frac{1}{2} + \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$
$= 2 + 2\sqrt{3}\boldsymbol{i}$

12.



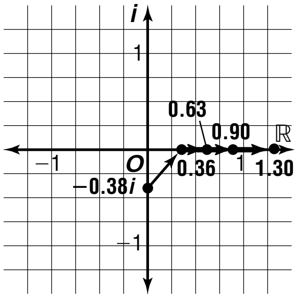
$2(\cos 3 + \boldsymbol{i}\sin 3)$
$\approx 2(-0.99 + \boldsymbol{i}(0.14))$
$= -1.98 + 0.28\boldsymbol{i}$

13.



$\frac{3}{2}(\cos 2\pi + \boldsymbol{i} \sin 2\pi)$
$= \frac{3}{2}(1 + \boldsymbol{i}(0))$
$= \frac{3}{2}$

14.



15a. magnitude $= \sqrt{10^2 + 15^2}$
$= \sqrt{325}$
≈ 18.03 N

15b. $\theta = \text{Arctan}\,\frac{15}{10}$
$\approx 56.31°$

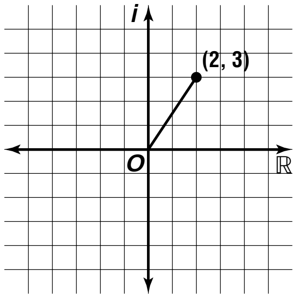
Pages 590–591 Exercises

16. $2x - 5y\boldsymbol{i} = 12 + 15\boldsymbol{i}$
$2x = 12 \qquad -5y = 15$
$x = 6 \qquad y = -3$

17. $1 + (x + y)\boldsymbol{i} = y + 3x\boldsymbol{i}$
$1 = y \qquad x + y = 3x$
$x + (1) = 3x$
$1 = 2x$
$\frac{1}{2} = x$

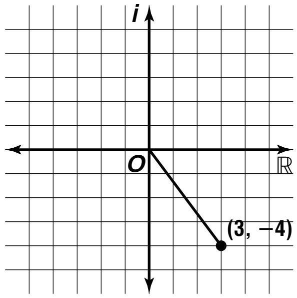
18. $4x + (y - 5)\boldsymbol{i} = 2x - y + (x + 7)\boldsymbol{i}$
$y - 5 = x + 7 \qquad 4x = 2x - y$
$y = x + 12 \qquad 4x = 2x - (x + 12)$
$3x = -12$
$x = -4$
$y = (-4) + 12$ or 8

19.



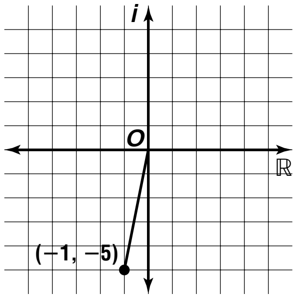
$|z| = \sqrt{2^2 + 3^2}$
$= \sqrt{13}$

20.



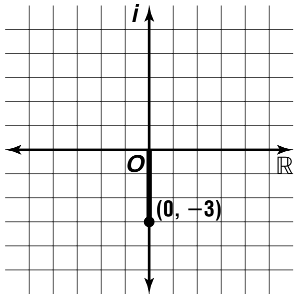
$|z| = \sqrt{3^2 + (-4)^2}$
$= \sqrt{25}$ or 5

21.



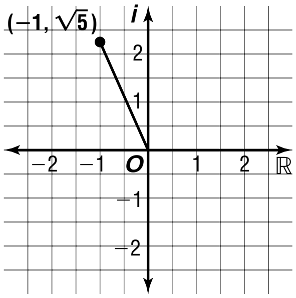
$|z| = \sqrt{(-1)^2 + (-5)^2}$
$= \sqrt{26}$

22.



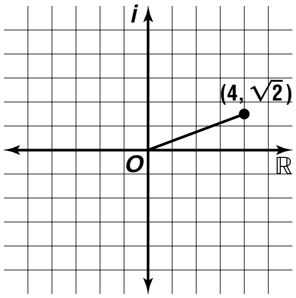
$|z| = \sqrt{0^2 + (-3)^2}$
$= \sqrt{9}$ or 3

23.



$|z| = \sqrt{(-1)^2 + (\sqrt{5})^2}$
$= \sqrt{6}$

24.



$|z| = \sqrt{4^2 + (\sqrt{2})^2}$
$= \sqrt{18}$ or $3\sqrt{2}$

25. $r = \sqrt{(-4)^2 + 6^2}$
$= \sqrt{52}$ or $2\sqrt{13}$

26. $r = \sqrt{3^2 + 3^2} \qquad \theta = \text{Arctan}\left(\frac{3}{3}\right)$
$= \sqrt{18}$ or $3\sqrt{2} \qquad = \frac{\pi}{4}$
$3 + 3\boldsymbol{i} = 3\sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$

27. $r = \sqrt{(-1)^2 + (-\sqrt{3})^2} \qquad \theta = \text{Arctan}\left(\frac{-\sqrt{3}}{-1}\right) + \pi$
$= \sqrt{4}$ or 2 $\qquad = \frac{4\pi}{3}$
θ is in the third quadrant.
$-1 - \sqrt{3}\boldsymbol{i} = 2\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$

28. $r = \sqrt{6^2 + (-8)^2} \qquad \theta = \text{Arctan}\left(\frac{-8}{6}\right) + 2\pi$
$= \sqrt{100}$ or 10 $\qquad \approx 5.36$
θ is in the fourth quadrant.
$6 - 8\boldsymbol{i} = 10(\cos 5.36 + \boldsymbol{i} \sin 5.36)$

29. $r = \sqrt{(-4)^2 + 1^2}$ $\qquad \theta = \text{Arctan}\left(\frac{1}{-4}\right) + \pi$

$= \sqrt{17}$ $\qquad \approx 2.90$

θ is in the second quadrant.

$-4 + \boldsymbol{i} = \sqrt{17}(\cos 2.90 + \boldsymbol{i} \sin 2.90)$

30. $r = \sqrt{20^2 + (-21)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-21}{20}\right) + 2\pi$

$= \sqrt{841}$ or 29 $\qquad \approx 5.47$

θ is in the fourth quadrant.

$20 - 21\boldsymbol{i} = 29(\cos 5.47 + \boldsymbol{i} \sin 5.47)$

31. $r = \sqrt{(-2)^2 + 4^2}$ $\qquad \theta = \text{Arctan}\left(\frac{4}{-2}\right) + \pi$

$= \sqrt{20}$ or $2\sqrt{5}$ $\qquad \approx 2.03$

θ is in the second quadrant.

$-2 + 4\boldsymbol{i} = 2\sqrt{5}(\cos 2.03 + \boldsymbol{i} \sin 2.03)$

32. $r = \sqrt{3^2 + 0^2}$ $\qquad \theta = \text{Arctan}\left(\frac{0}{3}\right)$

$= \sqrt{9}$ or 3 $\qquad = 0$

θ is on the x-axis at 3.

$3 = 3(\cos 0 + \boldsymbol{i} \sin 0)$

33. $r = \sqrt{(-4\sqrt{2})^2 + 0^2}$ $\qquad \theta = \text{Arctan}\,\frac{0}{-4\sqrt{2}} + \pi$

$= \sqrt{32}$ or $4\sqrt{2}$ $\qquad = \pi$

θ is on the x-axis at $-4\sqrt{2}$.

$-4\sqrt{2} = 4\sqrt{2}\,(\cos \pi + \boldsymbol{i} \sin \pi)$

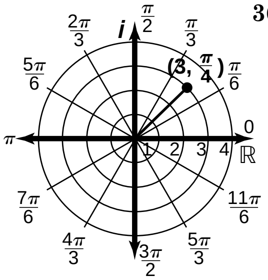
34. $r = \sqrt{0^2 + (-2)^2}$

$= \sqrt{4}$ or 2

Since $x = 0$ when $y = -2$, $\theta = \frac{3\pi}{2}$.

$-2\boldsymbol{i} = 2\left(\cos \frac{3\pi}{2} + \boldsymbol{i} \sin \frac{3\pi}{2}\right)$

35.

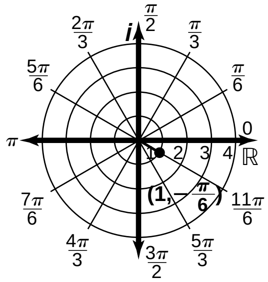


$3\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$

$= 3\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right)$

$= \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}\boldsymbol{i}$

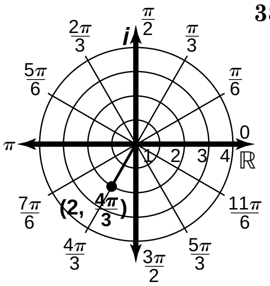
36.



$\cos\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}\boldsymbol{i}$

37.

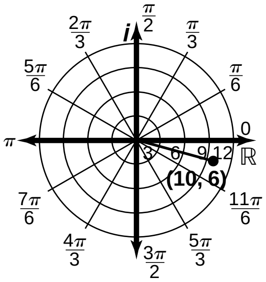


$2\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$

$= 2\left(-\frac{1}{2} - \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= -1 - \sqrt{3}\boldsymbol{i}$

38.

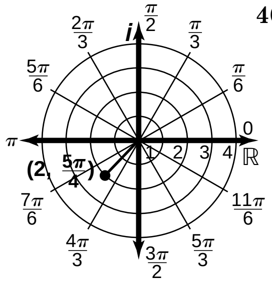


$10(\cos 6 + \boldsymbol{i} \sin 6)$

$\approx 10(0.960 + \boldsymbol{i}(-0.279))$

$= 9.60 - 2.79\boldsymbol{i}$

39.

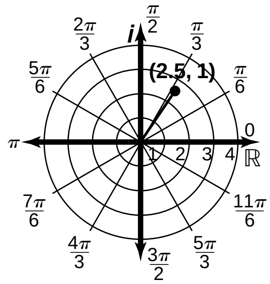


$2\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right)$

$= 2\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$= -\sqrt{2} - \sqrt{2}\boldsymbol{i}$

40.

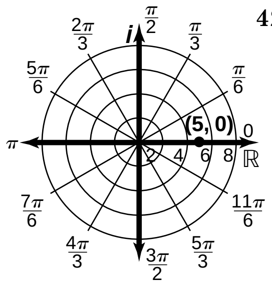


$2.5(\cos 1 + \boldsymbol{i} \sin 1)$

$\approx 2.5(0.54 + \boldsymbol{i}(0.84))$

$= 1.35 + 2.10\boldsymbol{i}$

41.

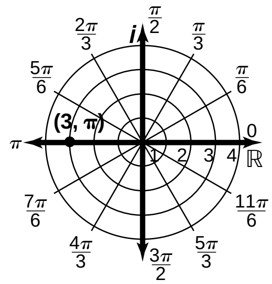


$5(\cos 0 + \boldsymbol{i} \sin 0)$

$= 5(1 + 0)$

$= 5$

42.

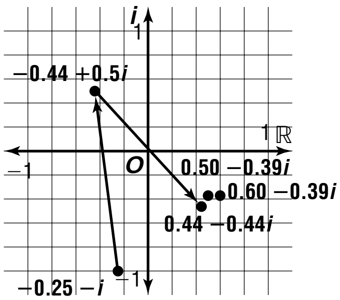


$3(\cos \pi + \boldsymbol{i} \sin \pi)$

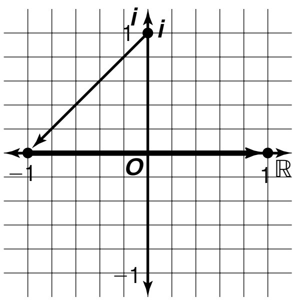
$= 3(-1 + 0)$

$= -3$

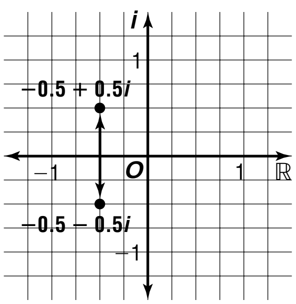
43.



44.



45.



46a. $40\angle 30° = 40(\cos 30° + \boldsymbol{j} \sin 30°)$
$= 40\left(\frac{\sqrt{3}}{2} + \boldsymbol{j}\left(\frac{1}{2}\right)\right)$
$= 34.64 + 20\boldsymbol{j}$
$60\angle 60° = 60(\cos 60° + \boldsymbol{j} \sin 60°)$
$= 60\left(\frac{1}{2} + \boldsymbol{j}\left(\frac{\sqrt{3}}{2}\right)\right)$
$= 30 + 51.96\boldsymbol{j}$

46b. $(34.64 + 20\boldsymbol{j}) + (30 + 51.96\boldsymbol{j})$
$= (34.64 + 30) + (20\boldsymbol{j} + 51.96\boldsymbol{j})$
$= 64.64 + 71.96\boldsymbol{j}$

46c. $v(t) = r \sin(250t + \theta°)$
$r = \sqrt{64.64^2 + 71.96^2}$ $\quad \theta = \text{Arctan } \frac{71.96}{64.64}$
≈ 96.73 $\quad \approx 48°$
$v(t) = 96.73 \sin(250t + 48°)$

47. The graph of the conjugate of a complex number is obtained by reflecting the original number about the real axis. This reflection does not change the modulus. Since the amplitude is reflected, we can write the amplitude of the conjugate as the opposite of the original amplitude. In other words, the conjugate of $r(\cos\theta + \boldsymbol{i} \sin\theta)$ can be written as $r(\cos(-\theta) + \boldsymbol{i} \sin(-\theta))$, or $r(\cos\theta - \boldsymbol{i} \sin\theta)$.

48a. $10(\cos 0.7 + \boldsymbol{j} \sin 0.7) \approx 7.65 + 6.44\boldsymbol{j}$
$16(\cos 0.5 + \boldsymbol{j} \sin 0.5) \approx 14.04 + 7.67\boldsymbol{j}$

48b. $(7.65 + 6.44\boldsymbol{j}) + (14.04 + 7.67\boldsymbol{j})$
$= (7.65 + 14.04) + (6.44\boldsymbol{j} + 7.67\boldsymbol{j})$
$= 21.69 + 14.11\boldsymbol{j}$ ohms

48c. $r = \sqrt{21.69^2 + 14.11^2}$ $\quad \theta = \text{Arctan } \frac{14.11}{21.69}$
≈ 25.88 $\quad \approx 0.58$
$21.69 + 14.11\boldsymbol{j} = 25.88(\cos 0.58 + \boldsymbol{j} \sin 0.58)$ ohms

49a. Translate 2 units to the right and down 3 units.

49b. Rotate 90° counterclockwise about the origin.

49c. Dilate by a factor of 3.

49d. Reflect about the real axis.

50a. Sample answer: let $z_1 = 1 + \boldsymbol{i}$ and $z_2 = 3 + 4\boldsymbol{i}$.
$z_1 z_2 = (1 + \boldsymbol{i})(3 + 4\boldsymbol{i})$
$= -1 + 7\boldsymbol{i}$

50b. $z_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + \boldsymbol{i} \sin\frac{\pi}{4}\right) \approx 1.41(\cos 0.79 + \boldsymbol{i} \sin 0.79)$
$z_2 = 5(\cos 0.93 + \boldsymbol{i} \sin 0.93)$
$z_1 z_2 = 5\sqrt{2}(\cos 1.71 + \boldsymbol{i} \sin 1.71)$
$= 7.07(\cos 1.71 + \boldsymbol{i} \sin 1.71)$

50c. Sample answer: Let $z_1 = 2 - 4\boldsymbol{i}$ and $z_2 = -1 + 3\boldsymbol{i}$. Then
$z_1 = 2\sqrt{5}(\cos 5.18 + \boldsymbol{i} \sin 5.18)$
$\approx 4.47(\cos 5.18 + \boldsymbol{i} \sin 5.18)$,
$z_2 = \sqrt{10}(\cos 1.89 + \boldsymbol{i} \sin 1.89)$
$\approx 3.16(\cos 1.89 + \boldsymbol{i} \sin 1.89)$, and
$z_1 z_2 = (2 - 4\boldsymbol{i})(-1 + 3\boldsymbol{i})$
$= 10 + 10\boldsymbol{i}$
$= 10\sqrt{2}\left(\cos\frac{\pi}{4} + \boldsymbol{i} \sin\frac{\pi}{4}\right)$
$= 14.14(\cos 0.79 + \boldsymbol{i} \sin 0.79)$.

50d. To multiply two complex numbers in polar form, multiply the moduli and add the amplitudes. (In the sample answer for 50c, note that 5.18 + 1.89 = 7.07, which is coterminal with 0.79.)

51. $(6 - 2\boldsymbol{i})(-2 + 3\boldsymbol{i}) = -12 + 22\boldsymbol{i} - 6\boldsymbol{i}^2$
$= -6 + 22\boldsymbol{i}$

52. $x = -3 \cos -135°$ $\quad y = -3 \sin -135°$
$= -3\left(-\frac{\sqrt{2}}{2}\right)$ $\quad = -3\left(-\frac{\sqrt{2}}{2}\right)$
$= \frac{3\sqrt{2}}{2}$ $\quad = \frac{3\sqrt{2}}{2}$
$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

53. magnitude $= \sqrt{(-3)^2 + 7^2}$
$= \sqrt{58}$
$\langle -3, 7\rangle = -3\vec{\boldsymbol{i}} + 7\vec{\boldsymbol{j}}$

54. $\tan 105° = \tan(60° + 45°)$
$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$
$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$
$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$
$= \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{}$
$= \frac{4 + 2\sqrt{3}}{-2}$
$= -2 - \sqrt{3}$

55. $w = \frac{\theta}{t}$
$= \frac{12(2\pi)}{1}$ or 24π
$v = r\omega$
$= 18(24\pi)$ or 432π cm/s
432π cm/s $= 4.32\pi$ m/s
≈ 13.57 m/s

56. $\sin A = \frac{12}{18}$
$A = \sin^{-1}\left(\frac{2}{3}\right)$
$A \approx 41.8°$

47. $\sqrt{2a - 1} = \sqrt{3a - 5}$
$2a - 1 = 3a - 5$
$4 = a$

58. as $x \to \infty$, $y \to \infty$; as $x \to -\infty$, $y \to \infty$

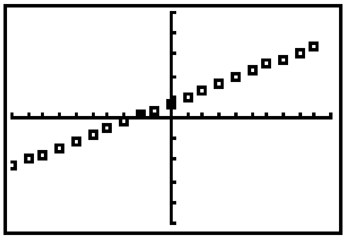
$y = 2x^2 + 2$	
x	y
-1000	2×10^6
-10	202
0	2
10	202
1000	2×10^6

59. In the fourth month, the person will have received 3 pay raises.
$\$500(1.10)^3 = \$665.50$
The correct choice is D.

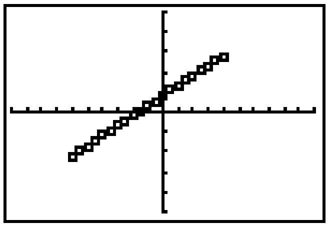
9-6B Graphing Calculator Exploration: Geometry in the Complex Plane

Page 592

1. They are collinear.



2. Yes. M is the point obtained when $T = 0$, and N is the point obtained when $T = 1$.
3. The points are again collinear, but closer together.



4. The points are on the line through M and N.
5. If one of a, b, or c equals 0, then $aK + bM + cN$ is on $\triangle KMN$. If none of a, b, or c equals 0, then $aK + bM + cN$ lies on or inside $\triangle KMN$.
6. M is the point obtained when $T = 0$ and N is the point obtained when $T = 1$. Thus, a point between M and N is obtained when $0 < T < 1$.
7. The distance between z and $1 - \boldsymbol{i}$ is 5. This defines a circle of radius 5 centered at $1 - \boldsymbol{i}$.
8. The distance between a point z and a point at $2 + 3\boldsymbol{i}$ is 2.
   $|z - (2 + 3\boldsymbol{i})| = 2$

9-7 Products and Quotients of Complex Numbers in Polar Form

Pages 596 Check for Understanding

1. The modulus of the quotient is the quotient of the moduli of the two complex numbers. The amplitude of the quotient is the difference of the amplitudes of the two complex numbers.
2. Square the modulus of the given complex number and double its amplitude.
3. Addition and subtraction are easier in rectangular form. Multiplication and division are easier in polar form. See students' work for examples.
4. $r = 2 \cdot 2$ or 4 $\qquad \theta = \frac{\pi}{2} + \frac{3\pi}{2} = \frac{4\pi}{2}$ or 2π

   $4(\cos 2\pi + \boldsymbol{i}\sin 2\pi) = 4(1 + \boldsymbol{i}(0)) = 4$

5. $r = \frac{3}{4}$ $\qquad \theta = \frac{\pi}{6} - \frac{2\pi}{3} = -\frac{3\pi}{6}$ or $-\frac{\pi}{2}$

   $\frac{3}{4}\left(\cos\left(-\frac{\pi}{2}\right) + \boldsymbol{i}\sin\left(-\frac{\pi}{2}\right)\right) = \frac{3}{4}(0 + (-1)\boldsymbol{i}) = -\frac{3}{4}\boldsymbol{i}$

6. $r = \frac{4}{2}$ or 2 $\qquad \theta = \frac{9\pi}{4} - \left(-\frac{\pi}{2}\right) = \frac{9\pi}{4} + \frac{2\pi}{4}$ or $\frac{11\pi}{4}$

   $2\left(\cos\frac{11\pi}{4} + \boldsymbol{i}\sin\frac{11\pi}{4}\right) = 2\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right) = -\sqrt{2} + \sqrt{2}\boldsymbol{i}$

7. $r = \frac{1}{2}(6)$ or 3 $\qquad \theta = \frac{\pi}{3} + \frac{5\pi}{6} = \frac{2\pi}{6} + \frac{5\pi}{6}$ or $\frac{7\pi}{6}$

   $3\left(\cos\frac{7\pi}{6} + \boldsymbol{i}\sin\frac{7\pi}{6}\right) = 3\left(-\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(-\frac{1}{2}\right)\right) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}\boldsymbol{i}$

8. $r_1 = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{16}$ or 4 $\qquad r_2 = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{12}$ or $2\sqrt{3}$

   $r = 4(2\sqrt{3})$ or $8\sqrt{3}$

   $\theta_1 = \text{Arctan}\left(\frac{2\sqrt{3}}{2}\right) = \frac{\pi}{3}$ $\qquad \theta_2 = \text{Arctan}\left(\frac{\sqrt{3}}{-3}\right) + \pi = \frac{5\pi}{6}$

   $\theta = \frac{\pi}{3} + \frac{5\pi}{6} = \frac{2\pi}{6} + \frac{5\pi}{6}$ or $\frac{7\pi}{6}$

   $8\sqrt{3}\left(\cos\frac{7\pi}{6} + \boldsymbol{i}\sin\frac{7\pi}{6}\right) = 8\sqrt{3}\left(-\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right) = -12 - 4\sqrt{3}\boldsymbol{i}$

9. $E = IZ = 2\left(\cos\frac{11\pi}{6} + \boldsymbol{j}\sin\frac{11\pi}{6}\right) \cdot 3\left(\cos\frac{\pi}{3} + \boldsymbol{j}\sin\frac{\pi}{3}\right)$

   $r = 2(3)$ or 6 $\qquad \theta = \frac{11\pi}{6} + \frac{\pi}{3} = \frac{11\pi}{6} + \frac{2\pi}{6} = \frac{13\pi}{6}$ or $\frac{\pi}{6}$

   $V = 6\left(\cos\frac{\pi}{6} + \boldsymbol{j}\sin\frac{\pi}{6}\right)$ volts

Pages 596–598 Exercises

10. $r = 4(7)$ or 28 $\qquad \theta = \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3}$ or π

    $28(\cos\pi + \boldsymbol{i}\sin\pi) = 28(-1 + \boldsymbol{i}(0)) = -28$

11. $r = \frac{6}{2}$ or 3 $\qquad \theta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4}$ or $\frac{\pi}{2}$

    $3\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right) = 3(0 + \boldsymbol{i}(1)) = 3\boldsymbol{i}$

12. $r = \frac{\frac{1}{2}}{3}$ or $\frac{1}{6}$ $\qquad \theta = \frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi}{6} - \frac{\pi}{6}$ or $\frac{\pi}{6}$

    $\frac{1}{6}\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right) = \frac{1}{6}\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{12} + \frac{1}{12}\boldsymbol{i}$

13. $r = 5(2)$ or 10

$$\begin{aligned}\theta &= \pi + \tfrac{3\pi}{4}\\ &= \tfrac{4\pi}{4} + \tfrac{3\pi}{4} \text{ or } \tfrac{7\pi}{4}\end{aligned}$$

$$\begin{aligned}10\left(\cos \tfrac{7\pi}{4} + \boldsymbol{i} \sin \tfrac{7\pi}{4}\right) &= 10\left(\tfrac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\tfrac{\sqrt{2}}{2}\right)\right)\\ &= 5\sqrt{2} - 5\sqrt{2}\boldsymbol{i}\end{aligned}$$

14. $r = 6(3)$ or 18

$$\begin{aligned}\theta &= -\tfrac{\pi}{3} + \tfrac{5\pi}{6}\\ &= -\tfrac{2\pi}{6} + \tfrac{5\pi}{6}\\ &= \tfrac{3\pi}{6} \text{ or } \tfrac{\pi}{2}\end{aligned}$$

$$\begin{aligned}18\left(\cos \tfrac{\pi}{2} + \boldsymbol{i} \sin \tfrac{\pi}{2}\right) &= 18(0 + \boldsymbol{i}(1))\\ &= 18\boldsymbol{i}\end{aligned}$$

15. $r = \frac{3}{1}$ or 3

$$\begin{aligned}\theta &= \tfrac{7\pi}{3} - \tfrac{\pi}{2}\\ &= \tfrac{14\pi}{6} - \tfrac{3\pi}{6} \text{ or } \tfrac{11\pi}{6}\end{aligned}$$

$$\begin{aligned}3\left(\cos \tfrac{11\pi}{6} + \boldsymbol{i} \sin \tfrac{11\pi}{6}\right) &= 3\left(\tfrac{\sqrt{3}}{2} + \boldsymbol{i}\left(-\tfrac{1}{2}\right)\right)\\ &= \tfrac{3\sqrt{3}}{2} - \tfrac{3}{2}\boldsymbol{i}\end{aligned}$$

16. $r = 2(3)$ or 6

$$\begin{aligned}\theta &= 240° + 60°\\ &= 300°\end{aligned}$$

$$\begin{aligned}6(\cos 300° + \boldsymbol{i} \sin 300°) &= 6\left(\tfrac{1}{2} + \boldsymbol{i}\left(-\tfrac{\sqrt{3}}{2}\right)\right)\\ &= 3 - 3\sqrt{3}\boldsymbol{i}\end{aligned}$$

17. $r = \dfrac{\sqrt{2}}{\frac{\sqrt{2}}{2}}$

$$\begin{aligned}\theta &= \tfrac{7\pi}{4} - \tfrac{3\pi}{4}\\ &= \tfrac{4\pi}{4} \text{ or } \pi\end{aligned}$$

$= \frac{2\sqrt{2}}{\sqrt{2}}$ or 2

$$\begin{aligned}2(\cos \pi + \boldsymbol{i} \sin \pi) &= 2(-1 + \boldsymbol{i}(0))\\ &= -2\end{aligned}$$

18. $r = 3(0.5)$ or 1.5 $\qquad \theta = 4 + 2.5$ or 6.5

$1.5(\cos 6.5 + \boldsymbol{i} \sin 6.5) \approx 1.46 + 0.32\boldsymbol{i}$

19. $r = \frac{4}{1}$ or 4 $\qquad \theta = -2 - 3.6$ or -5.6

$4[\cos(-5.6) + \boldsymbol{i} \sin(-5.6)] \approx 3.10 + 2.53\boldsymbol{i}$

20. $r = \frac{20}{15}$ or $\frac{4}{3}$

$$\begin{aligned}\theta &= \tfrac{7\pi}{6} - \tfrac{11\pi}{3}\\ &= \tfrac{7\pi}{6} - \tfrac{22\pi}{6}\\ &= -\tfrac{15\pi}{6} \text{ or } -\tfrac{\pi}{2}\end{aligned}$$

$$\begin{aligned}\tfrac{4}{3}\left(\cos -\tfrac{\pi}{2} + \boldsymbol{i} \sin -\tfrac{\pi}{2}\right) &= \tfrac{4}{3}(0 + \boldsymbol{i}(-1))\\ &= -\tfrac{4}{3}\boldsymbol{i}\end{aligned}$$

21. $r = 2(\sqrt{2})$ or $2\sqrt{2}$

$$\begin{aligned}\theta &= \tfrac{3\pi}{4} + \tfrac{\pi}{2}\\ &= \tfrac{3\pi}{4} + \tfrac{2\pi}{4} \text{ or } \tfrac{5\pi}{4}\end{aligned}$$

$$\begin{aligned}2\sqrt{2}\left(\cos \tfrac{5\pi}{4} + \boldsymbol{i} \sin \tfrac{5\pi}{4}\right) &= 2\sqrt{2}\left(-\tfrac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\tfrac{\sqrt{2}}{2}\right)\right)\\ &= -2 - 2\boldsymbol{i}\end{aligned}$$

22. $r = 2(6)$ or 12

$$\begin{aligned}\theta &= \tfrac{\pi}{3} + \left(-\tfrac{\pi}{6}\right)\\ &= \tfrac{2\pi}{6} - \tfrac{\pi}{6} \text{ or } \tfrac{\pi}{6}\end{aligned}$$

$$\begin{aligned}12\left(\cos \tfrac{\pi}{6} + \boldsymbol{i} \sin \tfrac{\pi}{6}\right) &= 12\left(\tfrac{\sqrt{3}}{2} + \boldsymbol{i}\left(\tfrac{1}{2}\right)\right)\\ &= 6\sqrt{3} + 6\boldsymbol{i}\end{aligned}$$

23. $r = \dfrac{4}{\frac{1}{2}}$ or 8

$$\begin{aligned}\theta &= \tfrac{5\pi}{3} - \tfrac{\pi}{3}\\ &= \tfrac{4\pi}{3}\end{aligned}$$

$$\begin{aligned}8\left(\cos \tfrac{4\pi}{3} + \boldsymbol{i} \sin \tfrac{4\pi}{3}\right) &= 8\left(-\tfrac{1}{2} + \boldsymbol{i}\left(-\tfrac{\sqrt{3}}{2}\right)\right)\\ &= -4 - 4\sqrt{3}\boldsymbol{i}\end{aligned}$$

24.

$$\begin{aligned}r_1 &= \sqrt{2^2 + (-2)^2} & r_2 &= \sqrt{(-3)^2 + 3^2}\\ &= \sqrt{8} \text{ or } 2\sqrt{2} & &= \sqrt{18} \text{ or } 3\sqrt{2}\end{aligned}$$

$r = 2\sqrt{2}(3\sqrt{2})$ or 12

$$\begin{aligned}\theta_1 &= \text{Arctan}\left(-\tfrac{2}{2}\right) + 2\pi & \theta_2 &= \text{Arctan}\left(\tfrac{3}{-3}\right) + \pi\\ &= -\tfrac{7\pi}{4} & &= \tfrac{3\pi}{4}\end{aligned}$$

$$\begin{aligned}\theta &= \tfrac{7\pi}{4} + \tfrac{3\pi}{4}\\ &= \tfrac{10\pi}{4} \text{ or } -\tfrac{\pi}{2}\end{aligned}$$

$$\begin{aligned}12\left(\cos \tfrac{\pi}{2} + \boldsymbol{i} \sin \tfrac{\pi}{2}\right) &= 12(0 + \boldsymbol{i}(1))\\ &= 12\boldsymbol{i}\end{aligned}$$

25.

$$\begin{aligned}r_1 &= \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} & r_2 &= \sqrt{(-3\sqrt{2})^2 + (-3\sqrt{2})^2}\\ &= \sqrt{4} \text{ or } 2 & &= \sqrt{36} \text{ or } 6\end{aligned}$$

$r = 2 \cdot 6$ or 12

$$\begin{aligned}\theta_1 &= \text{Arctan}\left(\tfrac{-\sqrt{2}}{\sqrt{2}}\right) + 2\pi & \theta_2 &= \text{Arctan}\left(\tfrac{-3\sqrt{2}}{-3\sqrt{2}}\right)\\ &= \tfrac{7\pi}{4} & &= \tfrac{5\pi}{4}\end{aligned}$$

$$\begin{aligned}\theta &= \tfrac{7\pi}{4} + \tfrac{5\pi}{4}\\ &= \tfrac{12\pi}{4} \text{ or } \pi\end{aligned}$$

$$\begin{aligned}12(\cos \pi + \boldsymbol{i} \sin \pi) &= 12(-1 + \boldsymbol{i}(0))\\ &= -12\end{aligned}$$

26.

$$\begin{aligned}r_1 &= \sqrt{(\sqrt{3})^2 + (-1)^2} & r_2 &= \sqrt{2^2 + (-2\sqrt{3})^2}\\ &= \sqrt{4} \text{ or } 2 & &= \sqrt{16} \text{ or } 4\end{aligned}$$

$r = \frac{2}{4}$ or $\frac{1}{2}$

$$\begin{aligned}\theta_1 &= \text{Arctan}\left(\tfrac{-1}{\sqrt{3}}\right) + 2\pi & \theta_2 &= \text{Arctan}\left(\tfrac{-2\sqrt{3}}{2}\right) + 2\pi\\ &= \tfrac{11\pi}{6} & &= \tfrac{5\pi}{3}\end{aligned}$$

$$\begin{aligned}\theta &= \tfrac{11\pi}{6} - \tfrac{5\pi}{3}\\ &= \tfrac{11\pi}{6} - \tfrac{10\pi}{6} \text{ or } \tfrac{\pi}{6}\end{aligned}$$

$$\begin{aligned}\tfrac{1}{2}\left(\cos \tfrac{\pi}{6} + \boldsymbol{i} \sin \tfrac{\pi}{6}\right) &= \tfrac{1}{2}\left(\tfrac{\sqrt{3}}{2} + \boldsymbol{i}\left(\tfrac{1}{2}\right)\right)\\ &= \tfrac{\sqrt{3}}{4} + \tfrac{1}{4}\boldsymbol{i}\end{aligned}$$

27.

$$\begin{aligned}r_1 &= \sqrt{(-4\sqrt{2})^2 + (4\sqrt{2})^2} & r_2 &= \sqrt{6^2 + 6^2}\\ &= \sqrt{64} \text{ or } 8 & &= \sqrt{72} \text{ or } 6\sqrt{2}\end{aligned}$$

$$\begin{aligned}r &= \tfrac{8}{6\sqrt{2}}\\ &= \tfrac{4}{3\sqrt{2}} \cdot \tfrac{\sqrt{2}}{\sqrt{2}}\\ &= \tfrac{4\sqrt{2}}{6} \text{ or } \tfrac{2\sqrt{2}}{3}\end{aligned}$$

$$\begin{aligned}\theta_1 &= \text{Arctan}\left(\tfrac{4\sqrt{2}}{-4\sqrt{2}}\right) + \pi & \theta_2 &= \text{Arctan}\left(\tfrac{6}{6}\right)\\ &= \tfrac{3\pi}{4} & &= \tfrac{\pi}{4}\end{aligned}$$

$$\begin{aligned}\theta &= \tfrac{3\pi}{4} - \tfrac{\pi}{4}\\ &= \tfrac{2\pi}{4} \text{ or } \tfrac{\pi}{2}\end{aligned}$$

$$\begin{aligned}\tfrac{2\sqrt{2}}{3}\left(\cos \tfrac{\pi}{2} + \boldsymbol{i} \sin \tfrac{\pi}{2}\right) &= \tfrac{2\sqrt{2}}{3}(0 + \boldsymbol{i}(1))\\ &= \tfrac{2\sqrt{2}}{3}\boldsymbol{i}\end{aligned}$$

28. $I = \frac{E}{Z}$

$= \frac{13}{3 - 2\boldsymbol{j}}$

$r_1 = 13$ $\qquad$ $r_2 = \sqrt{3^2 + (-2)^2}$
$= \sqrt{13}$

$r = \frac{13}{\sqrt{13}}$ or $\sqrt{13}$

$\theta_1 = 0$ $\qquad$ $\theta_2 = \text{Arctan}\left(-\frac{2}{3}\right)$
≈ -0.59

$\theta = 0 - (-0.59)$ or 0.59

$I = \sqrt{13}(\cos 0.59 + \boldsymbol{j} \sin 0.59) \approx 3 + 2\boldsymbol{j}$ amps

29. $Z = \frac{E}{I}$

$= \frac{100}{4 - 3\boldsymbol{j}}$

$r_1 = 100$ $\qquad$ $r_2 = \sqrt{4^2 + (-3)^2}$
$= \sqrt{25}$ or 5

$r = \frac{100}{5}$ or 20

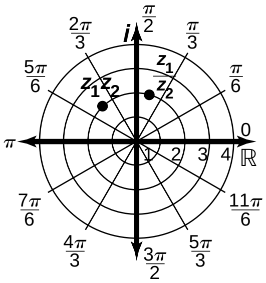
$\theta_1 = 0$ $\qquad$ $\theta_2 = \text{Arctan}\left(\frac{-3}{4}\right)$
≈ -0.64

$\theta = 0 - (-0.64)$ or 0.64

$z = 20(\cos 0.64 + \boldsymbol{j} \sin 0.64)$

$= 16 + 12\boldsymbol{j}$ ohms

30. Start at z_1 in the complex plane. Since the modulus of z_2 is 1, $z_1 z_2$ and $\frac{z_1}{z_2}$ will both have the same modulus as z_1. Then $z_1 z_2$ and $\frac{z_1}{z_2}$ can be located by rotating z_1 by $\frac{\pi}{6}$ counterclockwise and clockwise, respectively.



31a. The point is rotated counterclockwise about the origin by an angle of θ.

31b. The point is rotated 60° counterclockwise about the origin.

32. Since $a = 1$, the equation will be the form $z^2 + bz + c = 0$. The coefficient c is the product of the solutions, which is $6\left(\cos \frac{7\pi}{6} + \boldsymbol{i} \sin \frac{7\pi}{6}\right)$, or $-3\sqrt{3} - 3\boldsymbol{i}$ in rectangular form. The coefficient b is the opposite of the sum of the solutions, so convert the solutions to rectangular form to do the addition.

$b = -\left[3\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right) + 2\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right)\right]$

$= -\left[\left(\frac{3}{2} + \frac{3\sqrt{3}}{2}\boldsymbol{i}\right) + (-\sqrt{3} + \boldsymbol{i})\right]$

$= \left(-\frac{3}{2} + \sqrt{3}\right) + \left(\frac{-3\sqrt{3} - 2}{2}\right)\boldsymbol{i}$

Therefore, the equation is $z^2 + \left[\left(-\frac{3}{2} + \sqrt{3}\right) + \left(\frac{-3\sqrt{3} - 2}{2}\right)\boldsymbol{i}\right]z + (-3\sqrt{3} - 3\boldsymbol{i}) = 0$.

33. $r = \sqrt{5^2 + (-12)^2}$ $\qquad$ $\theta = \text{Arctan}\left(\frac{-12}{5}\right) + 2\pi$

$= \sqrt{169}$ or 13 $\qquad$ ≈ 5.11

$5 - 12\boldsymbol{i} = 13(\cos 5.11 + \boldsymbol{i} \sin 5.11)$

34. $r = 5 \sec\left(\theta - \frac{5\pi}{6}\right)$

$r \cos\left(\theta - \frac{5\pi}{6}\right) = 5$

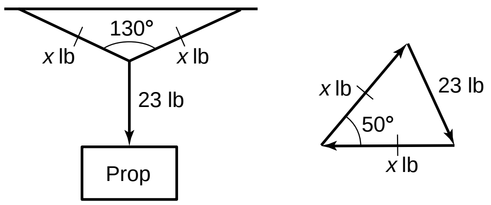
$r\left(\cos \theta \cos \frac{5\pi}{6} + \sin \theta \sin \frac{5\pi}{6}\right) - 5 = 0$

$-\frac{\sqrt{3}}{2} r \cos \theta + \frac{1}{2} r \sin \theta - 5 = 0$

$-\frac{\sqrt{3}}{2} x + \frac{1}{2} y - 5 = 0$

$-\sqrt{3}x + y - 10 = 0$

35.



Since the triangle is isosceles, the base angles are congruent. Each measures $\frac{180 - 50}{2}$ or 65°.

$\frac{23}{\sin 50°} = \frac{x}{\sin 65°}$

$23 \sin 65° = x \sin 50°$

$\frac{23 \sin 65°}{\sin 50°} = x$

$27.21 \approx x$; 27.21 lb

36. $\cos 2x + \sin x = 1$

$1 - 2\sin^2 x + \sin x = 1$

$2\sin^2 x - \sin x = 0$

$\sin x\,(2 \sin x - 1) = 0$

$\sin x = 0$ or $2 \sin x - 1 = 0$

$x = 0°$ $\qquad$ $\sin x = \frac{1}{2}$

$x = 30°$

37. $y = \cos x$

$x = \cos y$

$\arccos x = y$

38. $BC = ED = BE = AF = CD = 3$

$AB = FE = 2$

$AC = AB + BC$

$= 2 + 3$ or 5

$FD = FE + ED$

$= 2 + 3$ or 5

perimeter of rectangle $ACDF = 3 + 5 + 3 + 5$ or 16

perimeter of square $BCDE = 4(3)$ or 12

$16 - 12 = 4$

The correct choice is C.

9-8 Powers and Roots of Complex Numbers

Page 602 Graphing Calculator Exploration

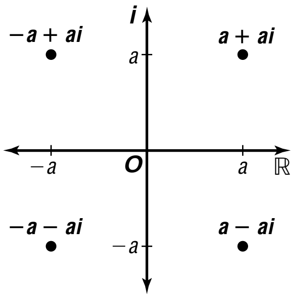
1. Rewrite 1 in polar form as $1(\cos 0 + \boldsymbol{i} \sin 0)$. Follow the keystrokes to find the roots at 1, $-0.5 + 0.87\boldsymbol{i}$, and $-0.5 - 0.87\boldsymbol{i}$.
2. Rewrite $\boldsymbol{i}$ in polar form as $1\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$.

   Follow the keystrokes to find the roots at $0.92 + 0.38\boldsymbol{i}$, $-0.38 + 0.92\boldsymbol{i}$, $-0.92 - 0.38\boldsymbol{i}$, and $0.38 - 0.92\boldsymbol{i}$.

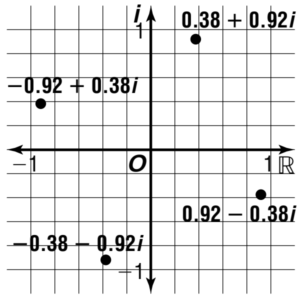
3. Rewrite $1 + \boldsymbol{i}$ in polar form as $\sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$.

   Follow the keystrokes to find the roots at $1.06 + 0.17\boldsymbol{i}$, $0.17 + 1.06\boldsymbol{i}$, $-0.95 + 0.49\boldsymbol{i}$, $-0.76 - -0.76\boldsymbol{i}$, and $0.49 - 0.95\boldsymbol{i}$.
4. equilateral triangle
5. regular pentagon
6. If $a > 0$ and $b = 0$, then $a + b\boldsymbol{i} = a$. The principal roots of a positive real number is a positive real number which would lie on the real axis in a complex plane.

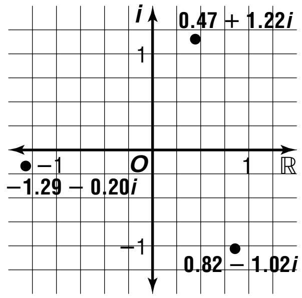
Pages 604-605 Check for Understanding

1. Same results, $-4 - 4\boldsymbol{i}$; answers may vary.
   $(1 + \boldsymbol{i})(1 + \boldsymbol{i})(1 + \boldsymbol{i})(1 + \boldsymbol{i})(1 + \boldsymbol{i})$
   $= (1 + 2\boldsymbol{i} + \boldsymbol{i}^2)(1 + 2\boldsymbol{i} + \boldsymbol{i}^2)(1 + \boldsymbol{i})$
   $= (2\boldsymbol{i})(2\boldsymbol{i})(1 + \boldsymbol{i})$
   $= -4(1 + \boldsymbol{i})$
   $= -4 - 4\boldsymbol{i}$
   $(1 + \boldsymbol{i})^5$
   $\rightarrow r = \sqrt{2}, \theta = \frac{\pi}{4}$
   $(\sqrt{2})^5 \left(\cos (5)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin (5)\left(\frac{\pi}{4}\right)\right)$
   $= 4\sqrt{2}\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right)$
   $= 4\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$
   $= -4 - 4\boldsymbol{i}$
2. Finding a reciprocal is the same as raising a number to the -1 power, so take the reciprocal of the modulus and multiply the amplitude by -1.
3. 

4. Shembala is correct. The polar form of $a + a\boldsymbol{i}$ is $a\sqrt{2}\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$. By De Moivre's Theorem, the polar form of $(a + a\boldsymbol{i})^2$ is $2a^2\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$. Since $\cos \frac{\pi}{2} = 0$, this is a pure imaginary number.
5. $r = \sqrt{(\sqrt{3})^2 + (-1)^2}$ or 2 $\quad \theta = \text{Arctan}\left(\frac{-1}{\sqrt{3}}\right)$ or $-\frac{\pi}{6}$
   $2^3\left(\cos (3)\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin (3)\left(-\frac{\pi}{6}\right)\right)$
   $= 8\left[\cos\left(-\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{2}\right)\right]$
   $= = 8(0 + \boldsymbol{i}\,(-1))$
   $= -8\boldsymbol{i}$
6. $r = \sqrt{3^2 + (-5)^2}$ or $\sqrt{34}$ $\quad \theta = \text{Arctan}\left(\frac{-5}{3}\right)$
   ≈ -1.030376827
   $\sqrt{34}^4 (\cos (4)(\theta) + \boldsymbol{i} \sin (4)(\theta))$
   $= -644 + 960\boldsymbol{i}$
7. $r = \sqrt{0^2 + 1^2}$ or 1 $\quad \theta = \frac{\pi}{2}$
   $1^{\frac{1}{6}}\left(\cos\left(\frac{1}{6}\right)\left(\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(\frac{1}{6}\right)\left(\frac{\pi}{2}\right)\right)$
   $= 1\left(\cos \frac{\pi}{12} + \boldsymbol{i} \sin \frac{\pi}{12}\right)$
   $\approx 0.97 + 0.26\boldsymbol{i}$
8. $r = \sqrt{(-2)^2 + (-1)^2}$ or $\sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{-1}{-2}\right) - \pi$
   ≈ -2.677945045
   $\sqrt{5}^{\frac{1}{3}}\left(\cos\left(\frac{1}{3}\right)(\theta) + \boldsymbol{i} \sin (3)(\theta)\right)$
   $= 0.82 - 1.02\boldsymbol{i}$
9. $x^4 + \boldsymbol{i} = 0 \rightarrow x^4 = -\boldsymbol{i}$
   Find the fourth roots of $-\boldsymbol{i}$.
   $r = \sqrt{0^2 + (-1)^2} = 1 \quad \theta = \frac{3\pi}{2}$
   $(-\boldsymbol{i})^{\frac{1}{4}} = \left[1\left(\cos\left(\frac{3\pi}{2} + 2n\pi\right) + \boldsymbol{i} \sin\left(\frac{3\pi}{2} + 2n\pi\right)\right)\right]^{\frac{1}{4}}$
   $= \cos \frac{3\pi + 4n\pi}{8} + \boldsymbol{i} \sin \frac{3\pi + 4n\pi}{8}$
   $x_1 = \cos \frac{3\pi}{8} + \boldsymbol{i} \sin \frac{3\pi}{8} \approx 0.38 + 0.92\boldsymbol{i}$
   $x_2 = \cos \frac{7\pi}{8} + \boldsymbol{i} \sin \frac{7\pi}{8} \approx -0.92 + 0.38\boldsymbol{i}$
   $x_3 = \cos \frac{11\pi}{8} + \boldsymbol{i} \sin \frac{11\pi}{8} \approx -0.38 - 0.92\boldsymbol{i}$
   $x_4 = \cos \frac{15\pi}{8} + \boldsymbol{i} \sin \frac{15\pi}{8} \approx 0.92 - 0.38\boldsymbol{i}$

10. $2x^3 + 4 + 2\boldsymbol{i} = 0 \rightarrow x^3 = -2 - \boldsymbol{i}$
   Find the third roots of $-2 - \boldsymbol{i}$.
   $r = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5} \quad \theta = \text{Arctan}\left(\frac{-1}{-2}\right) + \pi$
   ≈ 3.605240263
   $(-2 - \boldsymbol{i})^{\frac{1}{3}} = [\sqrt{5}(\cos (\theta + 2\boldsymbol{i}\pi) + \boldsymbol{i} \sin (\theta + 2n\pi))]^{\frac{1}{3}}$
   $= (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta + 2n\pi}{3} + \boldsymbol{i} \sin \frac{\theta + 2n\pi}{3}\right)$
   $x_1 = (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta}{3} + \boldsymbol{i} \sin \frac{\theta}{3}\right) = 0.47 + 1.22\boldsymbol{i}$
   $x_2 = (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta + 2\pi}{3} + \boldsymbol{i} \sin \frac{\theta + 2\pi}{3}\right) \approx -1.29 - 0.20\boldsymbol{i}$
   $x_3 = (\sqrt{5})^{\frac{1}{3}}\left(\cos \frac{\theta + 4\pi}{3} + \boldsymbol{i} \sin \frac{\theta + 4\pi}{3}\right) \approx 0.81 - 1.02\boldsymbol{i}$

11. For w_1, the modulus $= (\sqrt{0.8^2 + (-0.7)^2})^2$ or 1.13.
For w_2, the modulus $= 1.13^2$ or 1.28.
For w_3, the modulus $= 1.28^2$ or 1.64.
This moduli will approach infinity as the number of iterations increases. Thus, it is an escape set.

Pages 605–606 Exercises

12. $3^3\left(\cos (3)\left(\frac{\pi}{6}\right) + \boldsymbol{i} \sin (3)\left(\frac{\pi}{6}\right)\right)$
$= 27\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$
$= 27(0 + \boldsymbol{i}(1))$
$= 27\boldsymbol{i}$

13. $2^5\left(\cos (5)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin (5)\left(\frac{\pi}{4}\right)\right)$
$= 32\left(\cos \frac{5\pi}{4} + \boldsymbol{i} \sin \frac{5\pi}{4}\right)$
$= 32\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$
$= -16\sqrt{2} - 16\sqrt{2}\boldsymbol{i}$

14. $r = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$ $\quad \theta = \text{Arctan}\left(\frac{2}{-2}\right) = \frac{7\pi}{4}$
$(2\sqrt{2})^3\left(\cos (3)\left(\frac{7\pi}{9}\right) + \boldsymbol{i} \sin (3)\left(\frac{7\pi}{4}\right)\right)$
$= 16\sqrt{2}\left(\cos \frac{21\pi}{4} + \boldsymbol{i} \sin \frac{21\pi}{4}\right)$
$= 16\sqrt{2}\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right)$
$= 16 + 16\boldsymbol{i}$

15. $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$ $\quad \theta = \text{Arctan}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$
$2^4\left(\cos (4)\left(\frac{\pi}{3}\right) + \boldsymbol{i} \sin (4)\left(\frac{\pi}{3}\right)\right)$
$= 16\left(\cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3}\right)$
$= 16\left(-\frac{1}{2} + \boldsymbol{i}\left(-\frac{\sqrt{3}}{2}\right)\right)$
$= -8 - 8\sqrt{3}\boldsymbol{i}$

16. $r = \sqrt{3^2 + (-6)^2} = 3\sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{-6}{3}\right)$
≈ -1.107148718
$(3\sqrt{5})^4 (\cos (4)(\theta) + \boldsymbol{i} \sin (4)(\theta))$
$= -567 + 1944\boldsymbol{i}$

17. $r = \sqrt{2^2 + 3^2} = \sqrt{13}$ $\quad \theta = \text{Arctan}\left(\frac{3}{2}\right)$
≈ 0.9827937232
$(\sqrt{13})^{-2} (\cos (-2)(\theta) + \boldsymbol{i} \sin (-2)(\theta))$
$= -0.03 - 0.07\boldsymbol{i}$

18. $r = \sqrt{2^2 + 4^2} = 2\sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{4}{2}\right)$
$= 1.107148718$
$(2\sqrt{5})^4 (\cos (4)(\theta) + \boldsymbol{i} \sin (4)(\theta))$
$= -112 - 384\boldsymbol{i}$

19. $32^{\frac{1}{5}}\left(\cos \left(\frac{1}{5}\right)\left(\frac{2\pi}{3}\right) + \boldsymbol{i} \sin \left(\frac{1}{5}\right)\left(\frac{2\pi}{3}\right)\right)$
$= 2\left(\cos \frac{2\pi}{15} + \boldsymbol{i} \sin \frac{2\pi}{15}\right)$
$\approx 1.83 + 0.81\boldsymbol{i}$

20. $r = \sqrt{(-1)^2 + 0^2} = 1$ $\quad \theta = \pi$
$1^{\frac{1}{4}}\left(\cos \left(\frac{1}{4}\right)(\pi) + \boldsymbol{i} \sin \left(\frac{1}{4}\right)(\pi)\right)$
$= \cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}$
$\approx 0.71 + 0.71\boldsymbol{i}$

21. $r = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$ $\quad \theta = \text{Arctan}\left(\frac{1}{-2}\right) + \pi$
≈ 2.677945045
$(\sqrt{5})^{\frac{1}{4}}\left(\cos \left(\frac{1}{4}\right)(\theta) + \boldsymbol{i} \sin \left(\frac{1}{4}\right)(\theta)\right)$
$= 0.96 + 0.76\boldsymbol{i}$

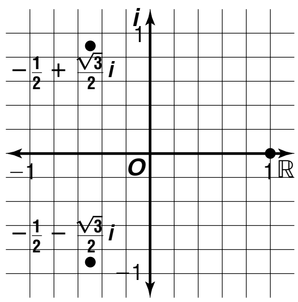
22. $r = \sqrt{4^2 + (-1)^2} = \sqrt{17}$ $\quad \theta = \text{Arctan}\left(\frac{-1}{4}\right)$
≈ -0.2449786631
$(\sqrt{17})^{\frac{1}{3}}\left(\cos \left(\frac{1}{3}\right)(\theta) + \boldsymbol{i} \sin \left(\frac{1}{3}\right)(\theta)\right)$
$= 1.60 - 0.13\boldsymbol{i}$

23. $r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ $\quad \theta = \text{Arctan}\left(\frac{2}{2}\right) = \frac{\pi}{4}$
$(2\sqrt{2})^{\frac{1}{3}}\left(\cos \left(\frac{1}{3}\right)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin \left(\frac{1}{3}\right)\left(\frac{\pi}{4}\right)\right)$
$= 1.37 + 0.37\boldsymbol{i}$

24. $r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$ $\quad \theta = \text{Arctan}\left(\frac{-1}{-1}\right) - \pi$
$= -\frac{3\pi}{4}$
$(\sqrt{2})^{\frac{1}{4}}\left(\cos \left(\frac{1}{4}\right)\left(-\frac{3\pi}{4}\right) + \boldsymbol{i} \sin \left(\frac{1}{4}\right)\left(-\frac{3\pi}{4}\right)\right)$
$= 0.91 - 0.61\boldsymbol{i}$

25. $r = \sqrt{0^2 + 1^2} = 1$ $\quad \theta = \frac{\pi}{2}$
$1^{\frac{1}{2}}\left(\cos \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right) + \boldsymbol{i} \sin \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)\right)$
$= 0.71 + 0.71\boldsymbol{i}$

26. $x^3 - 1 = 0 \rightarrow x^3 = 1$
Find the third roots of 1.
$r = \sqrt{1^2 + 0^2} = 1$ $\quad \theta = 0$
$1^{\frac{1}{3}} = [1 (\cos (0 + 2n\pi) + \boldsymbol{i} \sin (0 + 2n\pi))]^{\frac{1}{3}}$
$= \cos \frac{2n\pi}{3} + \boldsymbol{i} \sin \frac{2n\pi}{3}$
$x_1 = \cos 0 + \boldsymbol{i} \sin 0 = 1$
$x_2 = \cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}$
$x_3 = \cos \frac{4\pi}{3} + \boldsymbol{i} \sin \frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$



27. $x^5 + 1 \to x^5 = -1$

Find the fifth roots of -1.

$r = \sqrt{(-1)^2 + 0^2} = 1 \qquad \theta = \pi$

$(-1)^{\frac{1}{5}} = [1\,(\cos(\pi + 2n\pi) + \boldsymbol{i}\sin(\pi + 2n\pi))]^{\frac{1}{5}}$

$= \cos\frac{\pi + 2n\pi}{5} + \boldsymbol{i}\sin\frac{\pi + 2n\pi}{5}$

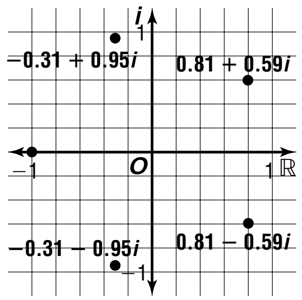
$x_1 = \cos\frac{\pi}{5} + \boldsymbol{i}\sin\frac{\pi}{5} = 0.81 + 0.59\boldsymbol{i}$

$x_2 = \cos\frac{3\pi}{5} + \boldsymbol{i}\sin\frac{3\pi}{5} = -0.31 + 0.95\boldsymbol{i}$

$x_3 = \cos\frac{5\pi}{5} + \boldsymbol{i}\sin\frac{5\pi}{5} = -1$

$x_4 = \cos\frac{7\pi}{5} + \boldsymbol{i}\sin\frac{7\pi}{5} = -0.31 - 0.95\boldsymbol{i}$

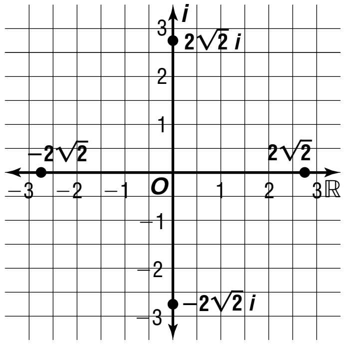
$x_5 = \cos\frac{9\pi}{5} + \boldsymbol{i}\sin\frac{9\pi}{5} = 0.81 - 0.59\boldsymbol{i}$



28. $2x^4 - 128 = 0 \to x^4 = 64$

Find the fourth roots of 64.

$r = \sqrt{64^2 + 0^2} = 64 \qquad \theta = 0$



$64^{\frac{1}{4}} = [64\,(\cos(0 + 2n\pi) + \boldsymbol{i}\sin(0 + 2n\pi))]^{\frac{1}{4}}$

$= 2\sqrt{2}\left(\cos\frac{n\pi}{2} + \boldsymbol{i}\sin\frac{n\pi}{2}\right)$

$x_1 = 2\sqrt{2}\,(\cos 0 + \boldsymbol{i}\sin 0) = 2\sqrt{2}$

$x_2 = 2\sqrt{2}\left(\cos\frac{\pi}{2} + \boldsymbol{i}\sin\frac{\pi}{2}\right) = 2\sqrt{2}\boldsymbol{i}$

$x_3 = 2\sqrt{2}\,(\cos\pi + \boldsymbol{i}\sin\pi) = -2\sqrt{2}$

$x_4 = 2\sqrt{2}\left(\cos\frac{3\pi}{2} + \boldsymbol{i}\sin\frac{3\pi}{2}\right) = -2\sqrt{2}\boldsymbol{i}$

29. $3x^4 + 48 = 0 \to x^4 = -16$

Find the fourth roots of -16.

$r = \sqrt{(-16)^2 + 0^2} = 16 \qquad \theta = \pi$

$(-16)^{\frac{1}{4}} = [16\,(\cos(\pi + 2n\pi) + \boldsymbol{i}\sin(\pi + 2n\pi))]^{\frac{1}{4}}$

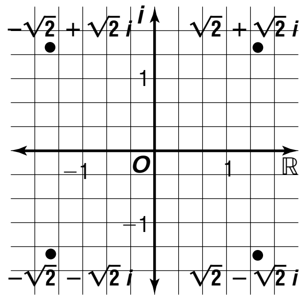
$= 2\left(\cos\frac{\pi + 2n\pi}{4} + \boldsymbol{i}\sin\frac{\pi + 2n\pi}{4}\right)$

$x_1 = 2\left(\cos\frac{\pi}{4} + \boldsymbol{i}\sin\frac{\pi}{4}\right) = \sqrt{2} + \sqrt{2}\boldsymbol{i}$

$x_2 = 2\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right) = -\sqrt{2} + \sqrt{2}\boldsymbol{i}$

$x_3 = 2\left(\cos\frac{5\pi}{4} + \boldsymbol{i}\sin\frac{5\pi}{4}\right) = -\sqrt{2} - \sqrt{2}\boldsymbol{i}$

$x_4 = 2\left(\cos\frac{7\pi}{4} + \boldsymbol{i}\sin\frac{7\pi}{4}\right) = \sqrt{2} - \sqrt{2}\boldsymbol{i}$



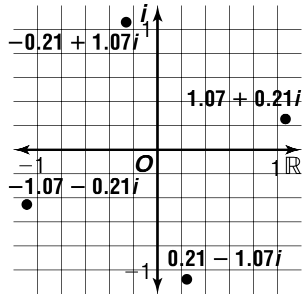
30. $x^4 - (1 + \boldsymbol{i}) = 0 \to x^4 = 1 + \boldsymbol{i}$

Find the fourth roots of $1 + \boldsymbol{i}$.

$r = \sqrt{1^2 + 1^2} = \sqrt{2} \qquad \theta = \text{Arctan}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$(1 + \boldsymbol{i})^{\frac{1}{4}} = \left[\sqrt{2}\left(\cos\left(\frac{\pi}{4} + 2n\pi\right) + \boldsymbol{i}\sin\left(\frac{\pi}{4} + 2n\pi\right)\right)\right]^{\frac{1}{4}}$

$= (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{\pi + 8n\pi}{16} + \boldsymbol{i}\sin\frac{\pi + 8n\pi}{16}\right)$



$x_1 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{\pi}{16} + \boldsymbol{i}\sin\frac{\pi}{16}\right) = 1.07 + 0.21\boldsymbol{i}$

$x_2 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{9\pi}{16} + \boldsymbol{i}\sin\frac{9\pi}{16}\right) = -0.21 + 1.07\boldsymbol{i}$

$x_3 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{17\pi}{16} + \boldsymbol{i}\sin\frac{17\pi}{16}\right) = -1.07 - 0.21\boldsymbol{i}$

$x_4 = (\sqrt{2})^{\frac{1}{4}}\left(\cos\frac{25\pi}{16} + \boldsymbol{i}\sin\frac{25\pi}{16}\right) = 0.21 - 1.07\boldsymbol{i}$

31. $2x^4 + 2 + 2\sqrt{3}\boldsymbol{i} = 0 \to x^4 = -1 - \sqrt{3}\boldsymbol{i}$.

Find the fourth roots of $-1 - \sqrt{3}\boldsymbol{i}$.

$r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$

$\theta = \text{Arctan}\left(\frac{-\sqrt{3}}{-1}\right) = \frac{\pi}{3} + \pi \text{ or } \frac{4\pi}{3}$

$(-1 - \sqrt{3}\boldsymbol{i})^{\frac{1}{4}}$

$= \left[2\left(\cos\left(\frac{4\pi}{3} + 2n\pi\right) + \boldsymbol{i}\sin\left(\frac{4\pi}{3} + 2n\pi\right)\right)\right]^{\frac{1}{4}}$

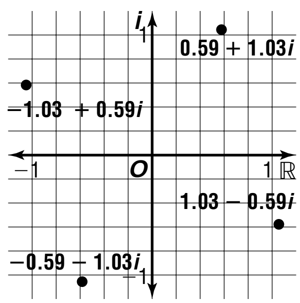
$= 2^{\frac{1}{4}}\left(\cos\frac{4\pi + 6n\pi}{12} + \boldsymbol{i}\sin\frac{4\pi + 6n\pi}{12}\right)$

$x_1 = 2^{\frac{1}{4}}\left(\cos\frac{4\pi}{12} + \boldsymbol{i}\sin\frac{4\pi}{12}\right) = 0.59 + 1.03\boldsymbol{i}$

$x_2 = 2^{\frac{1}{4}}\left(\cos\frac{10\pi}{12} + \boldsymbol{i}\sin\frac{10\pi}{12}\right) = -1.03 + 0.59\boldsymbol{i}$

$x_3 = 2^{\frac{1}{4}}\left(\cos\frac{22\pi}{12} + \boldsymbol{i}\sin\frac{22\pi}{12}\right) = -0.59 - 1.03\boldsymbol{i}$

$x_4 = 2^{\frac{1}{4}}\left(\cos\frac{28\pi}{12} + \boldsymbol{i}\sin\frac{28\pi}{12}\right) = 1.03 - 0.59\boldsymbol{i}$



32. Rewrite $10 - 9\boldsymbol{i}$ in polar form as
$\sqrt{181}\left[\cos\left(\tan^{-1}\left(\frac{-9}{10}\right)\right) + \boldsymbol{i}\sin\left(\tan^{-1}\left(\frac{-9}{10}\right)\right)\right]$.
Use a graphing calculator to find the fifth roots at $0.75 + 1.51\boldsymbol{i}$, $-1.20 + 1.18\boldsymbol{i}$, $-1.49 - 0.78\boldsymbol{i}$, $0.28 - 1.66\boldsymbol{i}$, and $1.66 - 0.25\boldsymbol{i}$.

33. Rewrite $2 + 4\boldsymbol{i}$ in polar form as
$2\sqrt{5}[\cos(\tan^{-1}(2)) + \boldsymbol{i}\sin(\tan^{-1}(2))]$.
Use a graphing calculator to find the sixth roots at $1.26 + 0.24\boldsymbol{i}$, $0.43 + 1.21\boldsymbol{i}$, $-0.83 + 0.97\boldsymbol{i}$, $-1.26 - 0.24\boldsymbol{i}$, $-0.43 - 1.21\boldsymbol{i}$, and $0.83 - 0.97\boldsymbol{i}$.

34. Rewrite $36 + 20\boldsymbol{i}$ in polar form as
$4\sqrt{106}\left[\cos\left(\tan^{-1}\left(\frac{5}{9}\right)\right) + \boldsymbol{i}\sin\left(\tan^{-1}\left(\frac{5}{9}\right)\right)\right]$.
Use a graphing calculator to find the eighth roots at $1.59 + 0.10\boldsymbol{i}$, $1.05 + 1.19\boldsymbol{i}$, $-0.10 + 1.59\boldsymbol{i}$, $-1.19 + 1.05\boldsymbol{i}$, $-1.59 - 0.10\boldsymbol{i}$, $-1.05 - 1.19\boldsymbol{i}$, $0.10 - 1.59\boldsymbol{i}$, and $1.19 - 1.05\boldsymbol{i}$.

35. For w_1, the modulus $= \left(\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2}\right)^2$ or 0.81.
For w_2, the modulus $= (0.81)^2$ or 0.66.
For w_3, the modulus $= (0.66)^2$ or 0.44.
This moduli will approach 0 as the number of iterations increase. Thus, it is a prisoner set.

36a. In polar form the 31st roots of 1 are given by $\cos\frac{2n\pi}{31} + \boldsymbol{i}\sin\frac{2n\pi}{31}$, $n = 0, 1, \ldots, 30$. Then $a = \cos\frac{2n\pi}{31}$. The maximum value of a cosine expression is 1, and it is achieved in this situation when $n = 0$.

36b. From the polar form in the solution to part a, we get $b = \sin\frac{2n\pi}{31}$. b will be maximized when $\frac{2n\pi}{31}$ is as close to $\frac{\pi}{2}$ as possible. This occurs when $n = 8$, so the maximum value of b is $\sin\frac{16\pi}{31}$, or about 0.9987.

37. $x^6 - 1 = 0 \to x^6 = 1$
Find the sixth roots of 1.
$r = \sqrt{1^2 + 0^2} = 1 \qquad \theta = 0$

$$1^{\frac{1}{6}} = [1(\cos(0 + 2n\pi) + \boldsymbol{i}\sin(0 + 2n\pi))]^{\frac{1}{6}}$$
$$= \cos\frac{n\pi}{3} + \boldsymbol{i}\sin\frac{n\pi}{3}$$

$x_1 = \cos 0 + \boldsymbol{i}\sin 0 = 1$

$x_2 = \cos\frac{\pi}{3} + \boldsymbol{i}\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}$

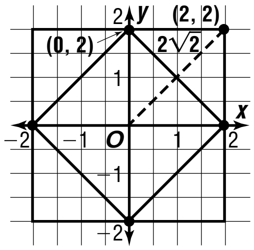
$x_3 = \cos\frac{2\pi}{3} + \boldsymbol{i}\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\boldsymbol{i}$

$x_4 = \cos\frac{3\pi}{3} + \boldsymbol{i}\sin\frac{3\pi}{3} = -1$

$x_5 = \cos\frac{4\pi}{3} + \boldsymbol{i}\sin\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$

$x_6 = \cos\frac{5\pi}{3} + \boldsymbol{i}\sin\frac{5\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$

38a. The point at (2, 2) becomes the point at (0, 2). From the origin, the point at (2, 2) had a length of $2\sqrt{2}$ and the new point at (0, 2) has a length of 2. The dilation factor is $\frac{\sqrt{2}}{2}$.



$$\frac{\sqrt{2}}{2}(\cos 45° + \boldsymbol{i}\sin 45°)$$
$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\sin\left(\frac{\sqrt{2}}{2}\right)\right)$$
$$= 0.5 + 0.5\boldsymbol{i}$$

38b. $\left[\frac{\sqrt{2}}{2}(\cos 45° + \boldsymbol{i}\sin 45°)\right]^2 = \frac{1}{2}(\cos 90° + \boldsymbol{i}\sin 90°)$
The square is rotated 90° counterclockwise and dilated by a factor of 0.5.

39. The roots are the vertices of a regular polygon. Since one of the roots must be a positive real number, a vertex of the polygon lies on the positive real axis and the polygon is symmetric about the real axis. This means that the non-real complex roots occur in conjugate pairs. Since the imaginary part of the sum of two complex conjugates is 0, the imaginary part of the sum of all the roots must be 0.

40. $r = 2(3)$ or 6 $\qquad \theta = \frac{\pi}{6} + \frac{5\pi}{3}$
$\qquad = \frac{\pi}{6} + \frac{10\pi}{6}$ or $\frac{11\pi}{6}$

$$6\left(\cos\frac{11\pi}{6} + \boldsymbol{i}\sin\frac{11\pi}{6}\right) = 6\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(-\frac{1}{2}\right)\right)$$
$$= 3\sqrt{3} - 3\boldsymbol{i}$$

41. $(2 - 5\boldsymbol{i}) + (-3 + 6\boldsymbol{i}) - (-6 + 2\boldsymbol{i})$
$= (2 + (-3) - (-6)) + (-5\boldsymbol{i} + 6\boldsymbol{i} - 2\boldsymbol{i})$
$= 5 - \boldsymbol{i}$

42. $x = t$, $y = -2t + 7$

43. $\cos 22.5° = \cos\left(\frac{45°}{2}\right)$

$$= \pm\sqrt{\frac{1 + \cos 45°}{2}}$$
$$= \pm\sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}}$$
$$= \pm\sqrt{\frac{2 + \sqrt{2}}{4}}$$
$$= \pm\frac{\sqrt{2 + \sqrt{2}}}{2}$$

44. Find B.
$B = 180° - 90° - 81°15'$
$= 8°45'$
Find a.

$$\tan 81°15' = \frac{a}{28}$$
$$28\tan 81°15' = a$$
$$181.9 = a$$

Find c.

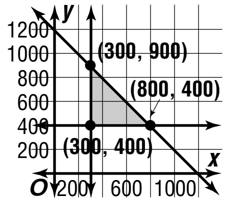
$\cos 81°15' = \frac{28}{c}$

$c = \frac{28}{\cos 81°15'}$

$c = 184.1$

45. Let x = the number of large bears produced.
Let y = the number of small bears produced.

$x \geq 300$
$y \geq 400$
$x + y \leq 1200$
$f(x, y) = 9x + 5y$
$f(300, 400) = 9(300) + 5(400)$
$= 4700$
$f(300, 900) = 9(300) + 5(900)$
$= 7200$
$f(800, 400) = 9(800) + 5(400)$
$= 9200$



Producing 800 large bears and 400 small bears yields the maximum profit.

46. $0.20(6) = 1.2$ quarts of alcohol
$0.60(4) = 2.4$ quarts of alcohol
$\frac{1.2 + 2.4}{6 + 4} = \frac{3.6}{10}$ or 36% alcohol
The correct choice is A.

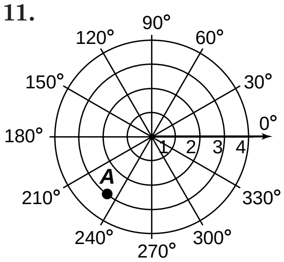
Chapter 9 Study Guide and Assessment

Pages 607 Check for Understanding

1. absolute value
2. Polar
3. prisoner
4. iteration
5. pure imaginary
6. cardioid
7. rectangular
8. spiral of Archimedes
9. Argand
10. modulus

Pages 608–610 Skills and Concepts

11.



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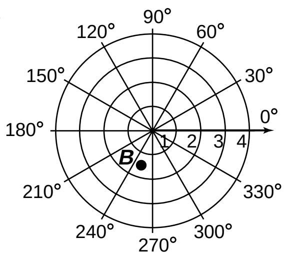

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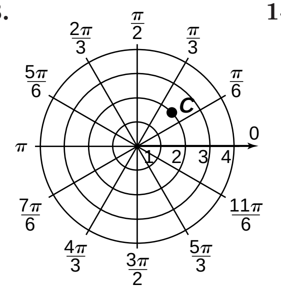

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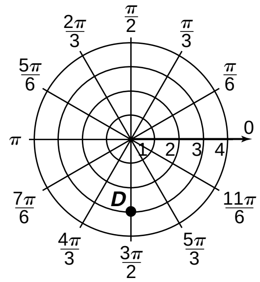
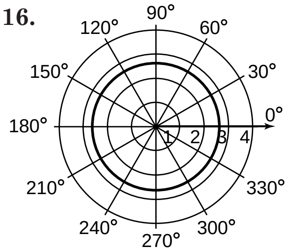


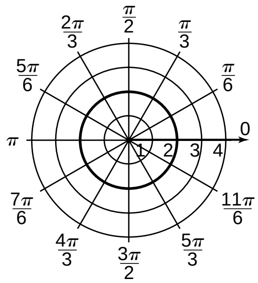
15. Sample answer: (4, 585°), (4, 945°), (−4, 45°), (−4, 405°)

$(r, \theta + 360k°)$
$\rightarrow (4, 225° + 360(1)°) \rightarrow (4, 585°)$
$\rightarrow (4, 225° + 360(2)°) \rightarrow (4, 945°)$
$(-r, \theta + (2k + 1)180°)$
$\rightarrow (-4, 225° + (-1)180°) \rightarrow (-4, 45°)$
$\rightarrow (-4, 225° + (1)180°) \rightarrow (-4, 405°)$

16.



17.



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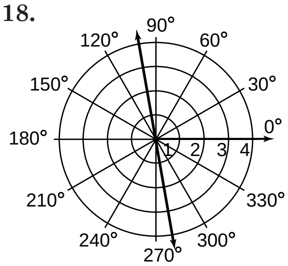

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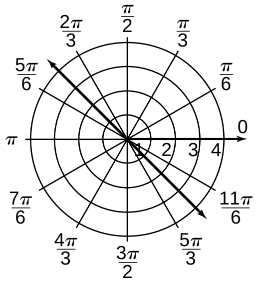

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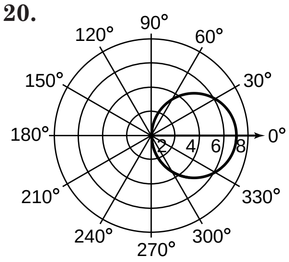
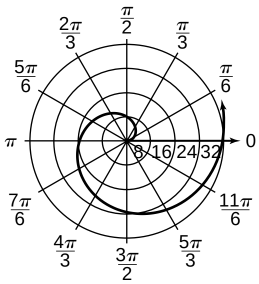


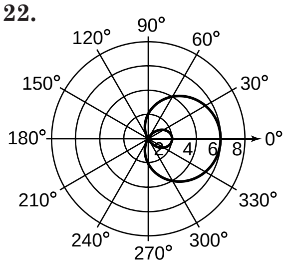
circle

21.



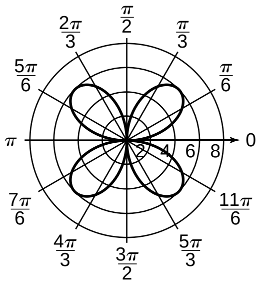
Spiral of Archimedes

22.



limaçon

23.



rose

24. $x = 6 \cos 45°$
$= 6\left(\frac{\sqrt{2}}{2}\right)$
$= 3\sqrt{2}$

$y = 6 \sin 45°$
$= 6\left(\frac{\sqrt{2}}{2}\right)$
$= 3\sqrt{2}$

$(3\sqrt{2}, 3\sqrt{2})$

25. $x = 2 \cos 330°$
$= 2\left(\frac{\sqrt{3}}{2}\right)$
$= \sqrt{3}$

$y = 2 \sin 330°$
$= 2\left(-\frac{1}{2}\right)$
$= -1$

$(\sqrt{3}, -1)$

26. $x = -2\cos\left(\frac{3\pi}{4}\right)$ $\qquad y = -2\sin\left(\frac{3\pi}{4}\right)$

$= -2\left(-\frac{\sqrt{2}}{2}\right)$ $\qquad = -2\left(\frac{\sqrt{2}}{2}\right)$

$= \sqrt{2}$ $\qquad = -\sqrt{2}$

$(\sqrt{2}, -\sqrt{2})$

27. $x = 1\cos\left(\frac{\pi}{2}\right)$ $\qquad y = 1\sin\left(\frac{\pi}{2}\right)$

$= 0$ $\qquad = 1$

$(0, 1)$

28. $r = \sqrt{(-\sqrt{3})^2 + (-3)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-3}{-\sqrt{3}}\right) + \pi$

$= \sqrt{12}$ or $2\sqrt{3}$ $\qquad = \frac{4\pi}{3}$

$\left(2\sqrt{3}, \frac{4\pi}{3}\right)$

29. $r = \sqrt{5^2 + 5^2}$ $\qquad \theta = \text{Arctan}\left(\frac{5}{5}\right)$

$= \sqrt{50}$ or $5\sqrt{2}$ $\qquad = \frac{\pi}{4}$

$\left(5\sqrt{2}, \frac{\pi}{4}\right)$

30. $r = \sqrt{(-3)^2 + 1^2}$ $\qquad \theta = \text{Arctan}\left(\frac{1}{-3}\right) + \pi$

$= \sqrt{10} \approx 3.16$ $\qquad \approx 2.82$

$(3.16, 2.82)$

31. $r = \sqrt{4^2 + 2^2}$ $\qquad \theta = \text{Arctan}\left(\frac{2}{4}\right)$

$= \sqrt{20} \approx 4.47$ $\qquad \approx 0.46$

$(4.47, 0.46)$

32. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{2^2 + 1^2}$

$= \pm\sqrt{5}$

Since C is positive, use $-\sqrt{5}$.

$-\frac{2}{\sqrt{5}}x - \frac{1}{\sqrt{5}}y - \frac{3}{\sqrt{5}} = 0$

$\cos\phi = -\frac{2\sqrt{5}}{5}$, $\sin\phi = -\frac{\sqrt{5}}{5}$, $p = \frac{3\sqrt{5}}{5}$

$\phi = \text{Arctan}\left(\frac{1}{2}\right)$

$\approx 27°$

Since $\cos\phi < 0$ and $\sin\phi < 0$, the normal lies in the third quadrant.

$\phi = 180° + 27°$ or $207°$

$p = r\cos(\theta - \phi)$

$\frac{3\sqrt{5}}{5} = r\cos(\theta - 207°)$

33. $\pm\sqrt{A^2 + B^2} = \pm\sqrt{3^2 + 1^2}$

$= \pm\sqrt{10}$

Since C is positive, use $-\sqrt{10}$.

$\frac{3}{\sqrt{10}}x - \frac{1}{\sqrt{10}}y - \frac{4}{\sqrt{10}} = 0$

$\cos\phi = -\frac{3\sqrt{10}}{10}$, $\sin\phi = -\frac{\sqrt{10}}{10}$, $p = \frac{2\sqrt{10}}{5}$

$\phi = \text{Arctan}\left(\frac{1}{3}\right)$

$\approx 18°$

Since $\cos\phi < 0$ and $\sin\phi < 0$, the normal lies in the third quadrant.

$\phi = 180° + 18°$ or $198°$

$p = r\cos(\theta - \phi)$

$\frac{2\sqrt{10}}{5} = r\cos(\theta - 198°)$

34. $3 = r\cos\left(\theta - \frac{\pi}{3}\right)$

$0 = r\cos\theta\cos\frac{\pi}{3} + r\sin\theta\sin\frac{\pi}{3} - 3$

$0 = \frac{1}{2}r\cos\theta + \frac{\sqrt{3}}{2}r\sin\theta - 3$

$0 = \frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3$

$0 = x + \sqrt{3}y - 6$ or

$x + \sqrt{3}y - 6 = 0$

35. $4 = r\cos\left(\theta + \frac{\pi}{2}\right)$

$0 = r\cos\theta\cos\frac{\pi}{2} - r\sin\theta\sin\frac{\pi}{2} - 4$

$0 = 0 - r\sin\theta - 4$

$0 = -y - 4$

$0 = y + 4$ or

$y + 4 = 0$

36. $\boldsymbol{i}^{10} + \boldsymbol{i}^{25} = (\boldsymbol{i}^4)^2 \cdot \boldsymbol{i}^2 + (\boldsymbol{i}^4)^6 \cdot \boldsymbol{i}$

$= (1)^2 \cdot (-1) + (1)^6 \cdot \boldsymbol{i}$

$= -1 + \boldsymbol{i}$

37. $(2 + 3\boldsymbol{i}) - (4 - 4\boldsymbol{i}) = (2 + (-4)) + (3\boldsymbol{i} - (-4\boldsymbol{i}))$

$= -2 + 7\boldsymbol{i}$

38. $(2 + 7\boldsymbol{i}) + (-3 - \boldsymbol{i}) = (2 + (-3)) + (7\boldsymbol{i} + (-\boldsymbol{i}))$

$= -1 + 6\boldsymbol{i}$

39. $\boldsymbol{i}^3(4 - 3\boldsymbol{i}) = 4\boldsymbol{i}^3 - 3\boldsymbol{i}^4$

$= 4(-\boldsymbol{i}) - 3(1)$

$= -3 - 4\boldsymbol{i}$

40. $(\boldsymbol{i} - 7)(-\boldsymbol{i} + 7) = -\boldsymbol{i}^2 + 14\boldsymbol{i} - 49$

$= 1 + 14\boldsymbol{i} - 49$

$= -48 + 14\boldsymbol{i}$

41. $\frac{4 + 2\boldsymbol{i}}{5 - 2\boldsymbol{i}} = \frac{4 + 2\boldsymbol{i}}{5 - 2\boldsymbol{i}} \cdot \frac{5 + 2\boldsymbol{i}}{5 + 2\boldsymbol{i}}$

$= \frac{20 + 18\boldsymbol{i} + 4\boldsymbol{i}^2}{25 - 4\boldsymbol{i}^2}$

$= \frac{16 + 18\boldsymbol{i}}{29}$

$= \frac{16}{29} + \frac{18}{29}\boldsymbol{i}$

42. $\frac{5 + \boldsymbol{i}}{1 - \sqrt{2}\boldsymbol{i}} = \frac{5 + \boldsymbol{i}}{1 - \sqrt{2}\boldsymbol{i}} \cdot \frac{1 + \sqrt{2}\boldsymbol{i}}{1 + \sqrt{2}\boldsymbol{i}}$

$= \frac{5 + 5\sqrt{2}\boldsymbol{i} + \boldsymbol{i} + \sqrt{2}\boldsymbol{i}^2}{1 - 2\boldsymbol{i}^2}$

$= \frac{(5 - \sqrt{2}) + (5\sqrt{2} + 1)\boldsymbol{i}}{3}$

$= \frac{5 - \sqrt{2}}{3} + \frac{1 + 5\sqrt{2}}{3}\boldsymbol{i}$

43. $r = \sqrt{2^2 + 2^2}$ $\qquad \theta = \text{Arctan}\left(\frac{2}{2}\right)$

$= \sqrt{8}$ or $2\sqrt{2}$ $\qquad = \frac{\pi}{4}$

$2\sqrt{2}\left(\cos\frac{\pi}{4} + \boldsymbol{i}\sin\frac{\pi}{4}\right)$

44. $r = \sqrt{1^2 + (-3)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-3}{1}\right) + 2\pi$

$= \sqrt{10}$ $\qquad \approx 5.03$

$\sqrt{10}(\cos 5.03 + \boldsymbol{i}\sin 5.03)$

45. $r = \sqrt{(-1)^2 + (\sqrt{3})^2}$ $\qquad \theta = \text{Arctan}\left(\frac{\sqrt{3}}{-1}\right) + \pi$

$= \sqrt{4}$ or 2 $\qquad = \frac{2\pi}{3}$

$2\left(\cos\frac{2\pi}{3} + \boldsymbol{i}\sin\frac{2\pi}{3}\right)$

46. $r = \sqrt{(-6)^2 + (-4)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-4}{-6}\right) + \pi$

$= \sqrt{52}$ or $2\sqrt{13}$ $\qquad \approx 3.73$

$2\sqrt{13}(\cos 3.73 + \boldsymbol{i}\sin 3.73)$

47. $r = \sqrt{(-4)^2 + (-1)^2}$ $\qquad \theta = \text{Arctan}\left(\frac{-1}{-4}\right) + \pi$

$= \sqrt{17}$ $\qquad \approx 3.39$

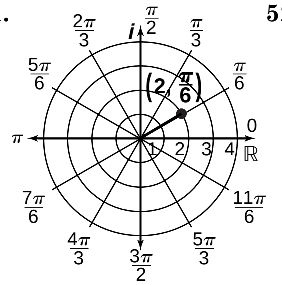
$\sqrt{17}(\cos 3.39 + \boldsymbol{i}\sin 3.39)$

48. $r = \sqrt{4^2 + 0^2}$ $\quad\theta = 0$
$= \sqrt{16}$ or 4
$4(\cos 0 + \boldsymbol{i} \sin 0)$

49. $r = \sqrt{(-2\sqrt{2})^2 + 0^2}$ $\quad\theta = \pi$
$= \sqrt{8}$ or $2\sqrt{2}$
$2\sqrt{2}(\cos \pi + \boldsymbol{i} \sin \pi)$

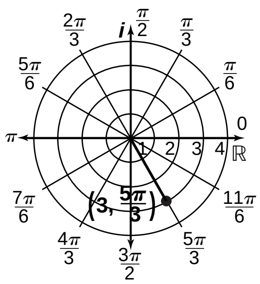
50. $r = \sqrt{0^2 + 3^2}$ $\quad\theta = \frac{\pi}{2}$
$= \sqrt{9}$ or 3
$3\left(\cos \frac{\pi}{2} + \boldsymbol{i} \sin \frac{\pi}{2}\right)$

51.



$2\left(\cos \frac{\pi}{6} + \boldsymbol{i} \sin \frac{\pi}{6}\right)$
$= 2\left(\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right)$
$= \sqrt{3} + \boldsymbol{i}$

52.



$3\left(\cos \frac{5\pi}{3} + \boldsymbol{i} \sin \frac{5\pi}{3}\right)$
$= 3\left(\frac{1}{2} + \boldsymbol{i}\left(-\frac{\sqrt{3}}{2}\right)\right)$
$= \frac{3}{2} - \frac{3\sqrt{3}}{2}\boldsymbol{i}$

53. $r = 4(3)$ or 12 $\quad\theta = \frac{\pi}{3} + \frac{\pi}{3}$ or $\frac{2\pi}{3}$
$12\left(\cos \frac{2\pi}{3} + \boldsymbol{i} \sin \frac{2\pi}{3}\right)$
$= 12\left(-\frac{1}{2} + \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$
$= -6 + 6\sqrt{3}\boldsymbol{i}$

54. $r = 8(4)$ or 32 $\quad\theta = \frac{\pi}{4} + \frac{\pi}{2}$
$\quad= \frac{\pi}{4} + \frac{2\pi}{4}$ or $\frac{3\pi}{4}$
$32\left(\cos \frac{3\pi}{4} + \boldsymbol{i} \sin \frac{3\pi}{4}\right)$
$= 32\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(\frac{\sqrt{2}}{2}\right)\right)$
$= -16\sqrt{2} + 16\sqrt{2}\boldsymbol{i}$

55. $r = 2(5)$ or 10 $\quad\theta = 2 + 0.5$ or 2.5
$10(\cos 2.5 + \boldsymbol{i} \sin 2.5)$
$\approx -8.01 + 5.98\boldsymbol{i}$

56. $r = \frac{8}{2}$ or 4 $\quad\theta = \frac{7\pi}{6} - \frac{5\pi}{3}$
$\quad= \frac{7\pi}{6} - \frac{10\pi}{6}$ or $-\frac{\pi}{2}$
$4\left[\cos\left(-\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{2}\right)\right]$
$= 4(0 + \boldsymbol{i}(-1))$
$= -4\boldsymbol{i}$

57. $r = \frac{6}{4}$ or $\frac{3}{2}$ $\quad\theta = \frac{\pi}{2} - \frac{\pi}{6}$
$\quad= \frac{3\pi}{6} - \frac{\pi}{6}$ or $\frac{\pi}{3}$
$\frac{3}{2}\left(\cos \frac{\pi}{3} + \boldsymbol{i} \sin \frac{\pi}{3}\right)$
$= \frac{3}{2}\left(\frac{1}{2} + \boldsymbol{i}\left(\frac{\sqrt{3}}{2}\right)\right)$
$= \frac{3}{4} + \frac{3\sqrt{2}}{4}\boldsymbol{i}$

58. $r = \frac{2.2}{4.4}$ or 0.5 $\quad\theta = 1.5 - 0.6$ or 0.9
$0.5(\cos 0.9 + \boldsymbol{i} \sin 0.9)$
$\approx 0.31 + 0.39\boldsymbol{i}$

59. $r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ $\quad\theta = \text{Arctan}\left(\frac{2}{2}\right) = \frac{\pi}{4}$
$(\sqrt{2})^8\left(\cos(8)\left(\frac{\pi}{4}\right) + \boldsymbol{i} \sin(8)\left(\frac{\pi}{4}\right)\right)$
$= 4096(\cos 2\pi + \boldsymbol{i} \sin 2\pi)$
$= 4096$

60. $r = (\sqrt{\sqrt{3})^2 + (-1)^2} = 2$
$\theta = \text{Arctan}\ \frac{-1}{\sqrt{3}} = -\frac{\pi}{6}$
$2^7\left(\cos(7)\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin(7)\left(-\frac{\pi}{6}\right)\right)$
$= 128\left(\cos -\frac{7\pi}{6} + \boldsymbol{i} \sin -\frac{7\pi}{6}\right)$
$= 128\left(-\frac{\sqrt{3}}{2} + \boldsymbol{i}\left(\frac{1}{2}\right)\right)$
$= -64\sqrt{3} + 64\boldsymbol{i}$

61. $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ $\quad\theta = \text{Arctan}\left(\frac{1}{-1}\right) + \pi$
$\quad= \frac{3\pi}{4}$
$(\sqrt{2})^4\left(\cos(4)\left(\frac{3\pi}{4}\right) + \boldsymbol{i} \sin(4)\left(\frac{3\pi}{4}\right)\right)$
$= 4(\cos 3\pi + \boldsymbol{i} \sin 3\pi)$
$= -4$

62. $r = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$ $\quad\theta = \text{Arctan}\left(\frac{-2}{-2}\right) + \pi$
$\quad= \frac{5\pi}{4}$
$(2\sqrt{2})^3\left(\cos(3)\left(\frac{5\pi}{4}\right) + \boldsymbol{i} \sin(3)\left(\frac{5\pi}{4}\right)\right)$
$= 16\sqrt{2}\left(\cos \frac{15\pi}{4} + \boldsymbol{i} \sin \frac{15\pi}{4}\right)$
$= 16\sqrt{2}\left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$
$= 16 - 16\boldsymbol{i}$

63. $r = \sqrt{0^2 + 1^2} = 1$ $\quad\theta = \frac{\pi}{2}$
$1^{\frac{1}{4}}\left(\cos\left(\frac{1}{4}\right)\left(\frac{\pi}{2}\right) + \boldsymbol{i} \sin\left(\frac{1}{4}\right)\left(\frac{\pi}{2}\right)\right)$
$= \cos \frac{\pi}{8} + \boldsymbol{i} \sin \frac{\pi}{8}$
$\approx 0.92 + 0.38\boldsymbol{i}$

64. $r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ $\quad\theta = \text{Arctan}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$
$2^{\frac{1}{3}}\left(\cos\left(\frac{1}{3}\right)\left(\frac{\pi}{6}\right) + \boldsymbol{i} \sin\left(\frac{1}{3}\right)\left(\frac{\pi}{6}\right)\right)$
$= 2^{\frac{1}{3}}\left(\cos\left(\frac{\pi}{18}\right) + \boldsymbol{i} \sin\left(\frac{\pi}{18}\right)\right)$
$\approx 1.24 + 0.22\boldsymbol{i}$

Page 611 Applications and Problem Solving

65. lemniscate

66. $r = \sqrt{75^2 + 125^2}$ $\quad\theta = \text{Arctan}\left(\frac{125}{75}\right)$
$= \sqrt{21{,}250} \approx 145.77$ $\quad\approx 59.04°$
$(145.77, 59.04°)$

67.
$$r\cos\left(\theta - \frac{\pi}{2}\right) + 5 = 0$$
$$r\cos\theta\cos\frac{\pi}{2} + r\sin\theta\sin\frac{\pi}{2} + 5 = 0$$
$$r\sin\theta + 5 = 0$$
$$y + 5 = 0$$
$$y = -5$$

68. $I = \frac{E}{Z}$
$= \frac{50 + 180\boldsymbol{j}}{4 + 5\boldsymbol{j}}$
$= \frac{50 + 180\boldsymbol{j}}{4 + 5\boldsymbol{j}} \cdot \frac{4 - 5\boldsymbol{j}}{4 - 5\boldsymbol{j}}$
$= \frac{200 + 470\boldsymbol{j} - 900\boldsymbol{j}^2}{16 - 25\boldsymbol{j}^2}$
$= \frac{1100 + 470\boldsymbol{j}^2}{41}$
$\approx 26.83 + 11.46\boldsymbol{j}$ amps

Page 611 Open-Ended Assessment

1a. Sample answer: $4 - 6\boldsymbol{i}$ and $3 + 2\boldsymbol{i}$

$$(4 - 6\boldsymbol{i}) + (3 + 2\boldsymbol{i}) = (4 + 3) + (-6\boldsymbol{i} + 2\boldsymbol{i})$$
$$= 7 - 4\boldsymbol{i}$$

1b. No. Sample explanation: $2 - 3\boldsymbol{i}$ and $5 - \boldsymbol{i}$ also have this sum.

$$(2 - 3\boldsymbol{i}) + (5 - \boldsymbol{i}) = (2 + 5) + (-3\boldsymbol{i} + (-\boldsymbol{i}))$$
$$= 7 - 4\boldsymbol{i}$$

2a. Sample answer: $4 + \boldsymbol{i}$

$$|z| = \sqrt{4^2 + 1^2}$$
$$= \sqrt{17}$$

2b. No. Sample explanation: $1 + 4\boldsymbol{i}$ also has this absolute value.

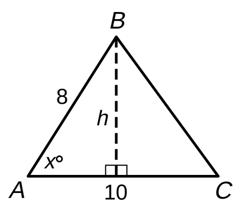
$$|z| = \sqrt{1^2 + 4^2}$$
$$= \sqrt{17}$$

Chapter 9 SAT & ACT Preparation

Page 613 SAT and ACT Practice

1. $\angle a$ and $\angle b$ form a linear pair, so $\angle b$ is supplementary to $\angle a$. Since $\angle b$ and $\angle d$ are vertical angles, they are equal in measure. So $\angle d$ is also supplementary to $\angle a$. Since $\angle d$ and $\angle f$ are alternate interior angles, they are equal. So $\angle f$ is supplementary to $\angle a$. And since $\angle f$ and $\angle h$ are vertical angles, $\angle h$ is supplementary to $\angle a$. The angles supplementary to $\angle a$ are angles b, d, f, and h. The correct choice is A.

2. Draw the given triangle and draw the height h from point B.

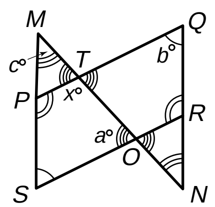


The answer choices include sin x. Write an expression for the height, using the sine of x.

$$\sin x = \frac{h}{8} \qquad A = \frac{1}{2}bh$$
$$8 \sin x = h \qquad = \frac{1}{2}(10)(8 \sin x)$$
$$= 40 \sin x$$

The correct choice is B.

3. Since $PQRS$ is a parallelogram, sides PQ and SR are parallel and $m\angle Q = m\angle S = b$.

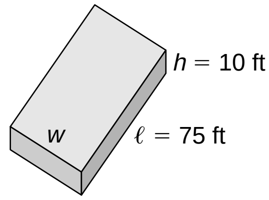


In $\triangle SMO$, $c + b + a = 180$ or $a = 180 - (c + b)$. Also, $x + a = 180$ or $a = 180 - x$ since consecutive interior angles are supplementary.

$$180 - (c + b) = 180 - x$$
$$x = c + b$$

The correct choice is E.

4. Volume $= \ell wh$



$$16{,}500 = 75 \cdot w \cdot 10$$
$$16{,}500 = 750w$$
$$22 = w$$

The correct choice is A.

5. $\frac{1}{100^{100}} - \frac{1}{10^{99}} = \frac{1}{100^{100}} - \frac{10}{10^{100}}$

$$= \frac{-9}{10^{100}}$$

The correct choice is A.

6. Consider the three unmarked angles at the intersection point. One of these angles, say the top one, is the supplement of the other two unmarked angles, because of vertical angles. So the sum of the measures of the unmarked angles is 180°.

The sum of the measures of the marked angles and the three unmarked angles is 3(180), since these angles are the interior angles of three triangles.

m(sum of marked angles) +
m(sum of unmarked angles) = 3(180)
m(sum of marked angles) + 180 = 3(180)
m(sum of marked angles) = 360

The correct choice is C.

7. Subtract the second equation from the first.

$$\begin{array}{rcl} 5x^2 + 6x & & = 70 \\ -5x^2 & + 6y & = 10 \\ \hline & 6x + 6y & = 60 \end{array}$$

$x + y = 10$, so $10x + 10y = 100$.

The correct choice is E.

8. Since $\angle B$ is a right angle, $\angle C$ is a right angle also, because they are alternate interior angles.

In the triangle containing $\angle C$, $90 + x + y = 180$ or $x + y = 90$.

The straight angle at D is made up of 3 angles.

$$120 + x + x = 180$$
$$2x = 60 \text{ or } x = 30$$
$$x + y = 90$$
$$(30) + y = 90$$
$$y = 60$$

The correct choice is B.

9. In the slope-intercept form of a line, $y = mx + b$, m represents the slope of the line, and b represents the y-intercept. Since the slope is given as $\frac{3}{2}$, the slope-intercept form of the line is $y = \frac{3}{2}x + b$.

Since (–3, 0) is on the line, it satisfies the equation. $0 = \frac{3}{2}(-3) + b$. So $b = \frac{9}{2}$.

The correct choice is D.

10. Note that consecutive interior angles are supplementary.

$$110 + 2x = 180 \qquad y + x = 180$$
$$2x = 70 \qquad y + (35) = 180$$
$$x = 35 \qquad y = 145$$

The answer is 145.